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STATISTICAL ANALYSIS OF PROGRESSIVE HYBRID TYPE-II CENCORED COMPETING RISS DATA

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ABSTRACT

In this article, we introduce a new scheme called progressive hybrid Type-II censoring scheme in the presence of competing risks (Type-II PHCS). Based on this scheme and assumed that the lifetimes of the failure times have an exponential distribution, the maximum likelihood and Bayes estimators of the distribution parameters are obtained, and the asymptotic confidence intervals and Bayes credible intervals are also proposed. In order to evaluate the performance of the estimators, a simulation study is carried out.

Keywords: ***Competing Risks, Type-II Progressive Hybrid Censoring, Exponential Distribution, Maximum Likelihood Estimation, Bayes Estimation, Lindley's Approximation, Asymptotic Confidence*** **Intervals**

1. INTRODUCTION:

In Reliability/engineering or medical sciences, generally observations are not completely known due to time and cost or inherent structure of the situations. Due to this cause censoring of the data can take place naturally. Therefore, the various censoring scheme can be classified by the experimenter depending upon the data obtaining processes. The two most common censoring schemes in life testing experiments are Type-I and Type-II censoring schemes and both censoring scheme have their own advantages whereas Type-II censoring scheme controls the efficiency of the test but time of the test is uncertain. The mixture of Type-I and Type-II censoring schemes, named as hybrid censoring scheme and this censoring scheme have been widely discussed in the literature [Epstein (1954)]. The hybrid censoring scheme is of two types namely Type-I hybrid and Type-II hybrid censoring scheme. Hybrid censored schemes have been introduced in the context of progressive censoring by Kundu and Joarder (2006) and Childs et al. (2008).

In Type-I progressive hybrid censoring scheme (Type-I PHCS) introduced by Kundu and Joarder (2006), can be described as follows. Suppose n identical items are put to the test and the life time distributions of the n items are denotes by $X_1, X_2, ..., X_n$. The integer $r < n$ is fixed at the beginning of the experiment, and $(R_1, R_2, \dots, R_r)$ are r per-fixed integers satisfying $R_1 + R_2 + \dots + R_r + r = n$. The time point is also fixed beforehand. At the time of the first failure $x_{(1)}$, R_1 of remaining (the $n-1$ surviving) units are randomly removed. Similarly, at the time of the second failure $x_{(2)}$, R_2 of the $n - R_1 - 2$ surviving units are removed, and so on. Finally at the time of the r^{th} failure all $R_r = n - R_1 - ... - R_{r-1} - r$ surviving units are removed from the life-test. In this type, the experiment would terminate at the random time $T^* = min\ (x_{(r)}, T)$.

Childs et al. (2008) introduced a Type-II progressive hybrid censoring scheme (Type-II PHCS). With the notation introduced in the preceding subsection, the Type-II PHCS involves the termination of the life test at time $T^* = max\ (x_{(r)}, T)$. Let D denote the number of failures that occur before time T, if

$X_{(r)} > T$, the experiment would terminate at the r^{th} failure, with the withdrawal of units occurring after each failure according to the pre-fixed progressive censoring scheme $(R_1, R_2, ..., R_r)$. However, if $X_{(r)} < T$, then instead of terminating the experiment by removing all remaining R_r units after the r^{th} failure, the experiment would continue to observe failures without any further withdrawals up to time T. Thus, in this case, $R_r = R_{r+1} = ... = R_D = 0$.

The main aim of this paper is analyzing the competing risk model when lifetimes have independent exponential distribution under Type-IIPHCS. We derive the maximum likelihood estimators (MLE) and Bayes estimators under squared error and LINEX loss functions using gamma priors. We also obtain the asymptotic confidence interval, credible interval and two bootstrap confidence intervals.

The rest of this paper is organized as follows: In section 2, we introduce the model and the notation used throughout this paper. In section 3, we discuss the maximum likelihood estimation. The Bayes estimators of the parameter under squared error and LINEX loss functions are derived in section 4. Different confidence intervals are presented in Section 5. In section 6, numerical illustration of the maximum likelihood and Bayes estimates and the corresponding confidence intervals are presented.

2. MODEL DESCRIPTION AND NOTATION:

In reliability analysis, the failure of items may be attributable to more than one cause at the same time. These "causes" are competing for the failure of the experimental unit. Consider a life time experiment with n identical units, where its lifetimes are described by independent and identically distributed (i.i.d) random variables $X_1, X_2, ..., X_n$. Without loss of generality; assume that there are only two causes of failure. We have $X_i = min\{X_{1i}, X_{2i}\}$ for $i = 1, ..., n,\ j = 1, 2,$ where X_{ji} denotes the latent failure time of the i^{th} unit under the j^{th} cause of failure. We assume that the latent failure times X_{1i} and X_{2i} are independent, and the pairs (X_{1i}, X_{2i}) are i.i.d. Assume that the failure time follows exponential distribution with cumulative distribution function $F_j(x)$ and failure hazard function $h_j(x)$ have the form

$$F_j(x) = 1 - e^{-\lambda_j x}, \quad h_j(x) = \lambda_j, \quad x \geq 0, \lambda_j > 0, \ j = 1,2. \tag{1}$$

Under Type-II progressive hybrid censoring scheme (Type-II PHCS) and in presence of competing risks data we have the following forms of observation:

Case I :

$$(X_{(1)}, c_1, R_1), (X_{(2)}, c_2, R_2), ..., (X_{(r)}, c_r, R_r) \quad if\ X_{(r)} \geq T,$$

Case II :

$$(X_{(1)}, c_1, R_1), ..., (X_{(D)}, c_D, R_D), (T, R_D^*) \quad if\ X_{(r)} < T.$$

where c_i is the indicator denoting the cause of failure, D denote the number of failures before time T and R_D^* is the number of remaining units left at the time point T with $R_r = R_{r+1} = ... = R_D = 0$. Further, we define

$$I_1(c_i = 1) = \begin{cases} 1, & c_i = 1 \\ 0 & else \end{cases}$$

and

$$I_2(c_i = 2) = \begin{cases} 1, & c_i = 2 \\ 0 & else \end{cases}$$

.

Thus, the random variables $r_1 = \sum_{i=1}^{r} I_1(c_i = 1)$ and $r_2 = \sum_{i=1}^{r} I_2(c_i = 2)$ describe the number of failures due to the first and the second cause of failures, respectively. In all procedures mentioned above, we assume that the cause of failure for all individuals to be known. Both r_1 and r_2 follow binomial distributions with sample size r. Using the independence of the latent failure times X_{1i}, X_{2i}, we can obtain the relative risk rate due to a particular cause (say, cause 1) as follows

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$$\pi_1 = P(X_{1i} < X_{2i}) = \int_0^\infty \lambda_1 e^{-\lambda_1 x_i} e^{-\lambda_2 x_i} dx = \frac{\lambda_1}{\lambda_1+\lambda_2}$$

similaly, $\pi_2 = P(X_{2i} < X_{1i}) = \dfrac{\lambda_2}{\lambda_1+\lambda_2}$.

For a given censoring scheme the likelihood function of the observed data when the cause of failure is known is given by

$$\text{Case I}: L(\lambda_1,\lambda_1)\alpha\prod_{i=1}^{r}(\left[f_1(x_i).\bar{F}_2(x_i)\right]^{I(c_i=1)} \left[f_2(x_i).\bar{F}_1(x_i)\right]^{I(c_i=2)}\left[\bar{F}_1(x_i).\bar{F}_2(x_i)\right]^{R_i}),$$

$$\text{Case II}: L(\lambda_1,\lambda_1)\alpha\prod_{i=1}^{r}(\left[f_1(x_i).\bar{F}_2(x_i)\right]^{I(c_i=1)} \left[f_2(x_i).\bar{F}_1(x_i)\right]^{I(c_i=2)}\left[\bar{F}_1(x_i).\bar{F}_2(x_i)\right]^{R_i}) \prod_{i=r+1}^{D}(\left[f_1(x_i).\bar{F}_2(x_i)\right]^{I(c_i=1)}\left[f_2(x_i).\bar{F}_1(x_i)\right]^{I(c_i=2)}) \left[\bar{F}_1(T).\bar{F}_2(T)\right]^{R_D^*}$$

Applying the identity $f_j(x) = h_j(x)\bar{F}_j(x)$, we can write the likelihood function as follows

$$\text{Case I}: L(\lambda_1,\lambda_1)\alpha\prod_{i=1}^{r}[h_1(x_i)]^{I(c_i=1)}\left[h_2(x_i)\right]^{I(c_i=2)} \left[\bar{F}_1(x_i).\bar{F}_2(x_i)\right]^{R_i+1}$$

$$\text{Case II}: L(\lambda_1,\lambda_1)\alpha\prod_{i=1}^{r}(\left[h_1(x_i)\right]^{I(c_i=1)} \left[h_2(x_i)\right]^{I(c_i=2)}\left[\bar{F}_1(x_i).\bar{F}_2(x_i)\right]^{R_i+1}) \prod_{i=r+1}^{D}(\left[h_1(x_i)\right]^{I(c_i=1)}\left[h_2(x_i)\right]^{I(c_i=2)} \left[\bar{F}_1(x_i).\bar{F}_2(x_i)\right]) \left[\bar{F}_1(T).\bar{F}_2(T)\right]^{R_D^*} \quad (2)$$

where $r = r_1 + r_2$, $D = D_1 + D_2$, $\bar{F}_j(x) = 1 - F_j(x)$ and $r, D > 0$.

3. MAXIMUM LIKELIHOOD ESTIMATION:

In this subsection, we have considered the maximum likelihood estimation of the parameters. Let us suppose that, $X_{(1)}, X_{(2)}, ..., X_{(n)}$ are n independent ordered lifetime failure observations in presence of Type-IIPHCS competing risks data from exponential distribution with parameters λ_j .From (1) and (2), the likelihood function can be rewritten as;

$$\text{Case I}: L(\lambda_1,\lambda_2) = \lambda_1^{r_1}\ \lambda_2^{r_2}\ e^{-(\lambda_1+\lambda_2)\sum_{i=1}^{r}(R_i+1)x_i}$$

$$\text{Case II}: L(\lambda_1,\lambda_2) = \lambda_1^{r_1+D_1}\lambda_2^{r_2+D_2} e^{-(\lambda_1+\lambda_2)\left[\sum_{i=1}^{r}(R_i+1)x_i + \sum_{i=r+1}^{D} x_i + T\ R_D^*\right]}$$

Thus combined likelihood can be written as;

$$L(\lambda_1,\lambda_2) = \lambda_1^{w_1}\ \lambda_2^{w_2}\ e^{-(\lambda_1+\lambda_2)U(x)} \quad (3)$$

where, w_j and $U(x_i)$ are defined as,

$$w_j = \begin{cases} \text{Case I}: r_j, \\ \text{Case II}: r_j + D_j \quad , j = 1,2. \end{cases}$$

and

$$U(x_i) = \begin{cases} \text{Case I}: \sum_{i=1}^{r}(R_i+1)x_i, \\ \text{Case II}: \sum_{i=1}^{r}(R_i+1)x_i + \sum_{i=r+1}^{D} x_i + T\ R_D^*. \end{cases}$$

Now, the Log likelihood function can be expressed as;

$$\ln L = w_1 \ln\lambda_1 + w_1 \ln\lambda_2 - (\lambda_1+\lambda_2)\ U(x_i) \quad (4)$$

upon differentiating (4) with respect to λ_1 and λ_2 we get the likelihood equations as

$$\frac{\partial \ln L}{\partial \lambda_1} = \frac{w_1}{\lambda_1} - U(x_i)$$

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and

$$\frac{\partial \ln L}{\partial \lambda_2} = \frac{w_2}{\lambda_2} - U(x_i) \qquad (5)$$

Equating the first derivations (5) to zero, we get the MLE of λ_1 and λ_2 in the following form

$$\hat{\lambda}_1 = \frac{w_1}{U(x_i)} \quad \text{and} \quad \hat{\lambda}_2 = \frac{w_2}{U(x_i)}.$$

From the log-likelihood function in (4), we have

$$\frac{\partial^2 \ln L}{\partial \lambda_1^2} = \frac{-w_1}{\lambda_1^2}, \qquad \frac{\partial^2 \ln L}{\partial \lambda_2^2} = \frac{-w_2}{\lambda_2^2}$$

$$\text{and} \quad \frac{\partial^2 \ln L}{\partial \lambda_1 \lambda_2} = 0. \qquad (6)$$

The Fisher information matrix $I(\lambda_1, \lambda_2)$ is then obtained by taking the expectation of minus equations (6), this expectation is difficult to obtained, so, under some regularity conditions, $(\hat{\lambda}_1, \hat{\lambda}_2)$ is approximately bivariately normal with mean (λ_1, λ_2) and covariance matrix $I^{-1}(\lambda_1, \lambda_2)$ [Cohen (1965)]. Practically, we $I^{-1}(\lambda_1, \lambda_2)$ by $I^{-1}(\hat{\lambda}_1, \hat{\lambda}_2)$, then

$$I^{-1}(\lambda_1,\lambda_2) = \begin{bmatrix} -\frac{\partial^2 \ln L}{\partial \lambda_1^2} & -\frac{\partial^2 \ln L}{\partial \lambda_1 \partial \lambda_2} \\ -\frac{\partial^2 \ln L}{\partial \lambda_2 \partial \lambda_1} & -\frac{\partial^2 \ln L}{\partial \lambda_2^2} \end{bmatrix}^{-1}_{(\lambda_1 = \hat{\lambda}_1, \lambda_2 = \hat{\lambda}_2)} = \begin{bmatrix} \mathrm{var}(\lambda_1) & 0 \\ 0 & \mathrm{var}(\lambda_2) \end{bmatrix}$$

this leads to

$$\mathrm{Var}(\hat{\lambda}_1) = \hat{\lambda}_1^2 / w_1, \ \mathrm{Var}(\hat{\lambda}_2) = \hat{\lambda}_2^2 / w_2$$

and $Cov(\hat{\lambda}_1, \hat{\lambda}_2) = 0$.

The MLE of the survival functions due to causes 1 and 2 are

$$\hat{\bar{F}}_1(x) = e^{-\hat{\lambda}_1 . x} \quad \text{and} \quad \hat{\bar{F}}_2(x) = e^{-\hat{\lambda}_2 . x}$$

Using the independence of the latent failure times X_{1i}, X_{2i}, $i = 1,\dots,n$, Kundu and

Joarder (2006) obtain the relative risk rate due to cause 1as follows

$$\pi_1 = P(X_{1i} \le X_{2i}) = \int_0^\infty \lambda_1 e^{-\lambda_1 x} e^{-\lambda_2 x} .dx = \frac{\lambda_1}{\lambda_1 + \lambda_2}$$

and because of the invariance property of the MLE, the relative risk rate $\hat{\pi}_1$ due to cause 1 is

$$\hat{\pi}_1 = \frac{\hat{\lambda}_1}{\hat{\lambda}_1 + \hat{\lambda}_2}$$

similarly,

$$\pi_2 = P(X_{2i} \le X_{1i}) = \frac{\hat{\lambda}_2}{\hat{\lambda}_1 + \hat{\lambda}_2}.$$

4. BAYESIAN ESTIMATION:

We consider the Bayesian estimation under the assumption that the random variables λ_j, $j = 1, 2$, has a gamma prior distribution with known shape and scale parameter a_j and b_j, with pdf given by

$$g(\lambda_j) \alpha \frac{b_j^{a_j}}{\Gamma(a_j)} . \lambda_j^{a_j - 1} e^{-b_j \lambda_j} \ , \ \lambda_j > 0, a_j, b_j > 0. \qquad (7)$$

combining (3) and (7), the joint posterior density of λ_1 and λ_2 given the data is

$$g(\lambda_1, \lambda_2 \backslash \underline{x}) = \frac{1}{A} \lambda_1^{a_1 + w_1 - 1} \lambda_2^{a_2 + w_2 - 1} e^{-[\lambda_1 (b_1 + U(x)) + \lambda_2 (b_2 + U(x))]}$$

where

$$A1 = \frac{\Gamma(a_1 + w_1)\Gamma(a_2 + w_2)}{(b_1 + U(x_i))^{a_1 + w_1} . (b_2 + U(x_i))^{a_2 + w_2}} \text{ is}$$

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the normalized constant. The marginal posterior of λ_1 and λ_2 can be obtained as follows

$$g(\lambda_j \mid \underline{x}) = \frac{\left(b_j + U(x_i)\right)^{a_j + w_j}}{\Gamma(a_j + w_j)} \lambda_j^{a_j + w_j - 1} \exp\left\{-\lambda_j \left[b_j + U(x_i)\right]\right\},\ j = 1, 2.$$

It is clear that $g(\lambda_1 \mid \underline{x})$ is the density function of a gamma $\left(a_1 + w_1, b_1 + U(x_i)\right)$ random variable, and $g(\lambda_2 \mid \underline{x})$ is the density function of a gamma $\left(a_2 + w_2, b_2 + U(x_i)\right)$ random variable. Under SE loss function and from the marginal posterior of λ_1 and λ_2 the Bayes estimator of λ_1 and λ_2 is the posterior mean which obtained as follows

$$\tilde{\lambda}_{SE\ j} = \frac{a_j + w_j}{b_j + U(x_i)}, \quad j = 1, 2.$$

The Bayes risk associated with λ_1 and λ_2 can be obtained as follows

$$\Re(\lambda_1) = E\left(\lambda_1^2\right) - \left[E\left(\lambda_1\right)\right]^2$$

$$\Re(\lambda_j) = \frac{(a_j + w_j + 2)}{\left(b_j + U(x_i)\right)^2}, \quad j = 1, 2.$$

where

$$E\left(\lambda_j^h\right) = \frac{\Gamma(a_j + w_j + \mathrm{h})}{\Gamma(a_j + w_j).\left(b_j + U(x_i)\right)^h},\ h = 1, 2, \ldots$$

is the marginal posterior h^{th} moments of λ_1. Under asymmetric LINEX loss function

$$\ell(\tilde{\lambda}_{Lin}, \lambda) = e^{c(\tilde{\lambda}_{Lin\ j} - \lambda_j)} - c(\tilde{\lambda}_{Lin\ j} - \lambda_j) - 1,$$

and from the marginal posterior of λ_1 and λ_2, the Bayes estimator of λ_1 and λ_2 can be obtained by minimizing the expected loss as follows

$$\frac{d}{d\tilde{\lambda}_{Lin\ j}} \int_0^\infty \left\{e^{c(\tilde{\lambda}_{Lin\ j} - \lambda_1)} - c(\tilde{\lambda}_{Lin\ j} - \lambda_j) - 1\right\} g(\lambda_j \mid \underline{x}).d\lambda_j = 0$$

differentiation gives

$$\int_0^\infty \left\{c.e^{c(\tilde{\lambda}_{Lin\ j} - \lambda_j)} - c\right\}.g(\lambda_j \mid \underline{x}).d\lambda_j = 0$$

$$\int_0^\infty e^{c(\tilde{\lambda}_{Lin\ j} - \lambda_j)}.g(\lambda_j \mid \underline{x}).d\lambda_j = 1$$

$$e^{c.\tilde{\lambda}_{Lin\ j}}.\int_0^\infty e^{-c.\lambda_j}.g(\lambda_j \mid \underline{x}).d\lambda_j = 1$$

integration gives

$$e^{c.\tilde{\lambda}_{Lin\ j}}.E(e^{-c.\lambda_j}) = 1$$

taking logarithm of both sides

$$c.\tilde{\lambda}_{Lin\ j} + \ln E(e^{-c.\lambda_j}) = 0$$

$$\therefore \tilde{\lambda}_{Lin\ j} = \frac{-1}{c}.\ln E(e^{-c.\lambda_j})$$

$$= \frac{-(a_j + w_j)}{c} \ln\left\{\frac{b_j + U(x_i)}{c + b_j + U(x_i)}\right\},\quad c \neq 0, \quad j = 1, 2.$$

where

$$E(e^{-c.\lambda_j}) = \left(\frac{b_j + U(x_i)}{\left(c + b_j + U(x_i)\right)}\right)^{(a_j + w_j)},\quad j = 1, 2.$$

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Under LINEX loss function we can obtain the Bayes risk associated with λ_1 and λ_2 by obtaining the expected loss as follows

$$\Re(\lambda_j) = \int_0^\infty \left\{ e^{c(\tilde{\lambda}_{Lin\,j} - \lambda_j)} - c(\tilde{\lambda}_{Lin\,j} - \lambda_j) - 1 \right\}.$$

$$g(\lambda_j \mid \underline{x}).d\,\lambda_j = e^{c.\tilde{\lambda}_{Lin\,j}} \int_0^\infty e^{-c.\lambda_j}.g(\lambda_j \mid \underline{x}).d\,\lambda_j$$

$$-c.\tilde{\lambda}_{Lin\,j} + c.\int_0^\infty \lambda_j.g(\lambda_j \mid \underline{x}).d\,\lambda_j.-1$$

$$= e^{-\ln E(e^{-c.\lambda_j})}.E(e^{-c.\lambda_j}) - \ln E(e^{-c.\lambda_j}) + c.\tilde{\lambda}_j.-1$$

$$= -E(e^{-c.\lambda_j})^2 - \ln E(e^{-c.\lambda_j}) + c.\tilde{\lambda}_j - 1$$

then,

$$\Re(\lambda_j) = -\left(\frac{b_j + U(x_i)}{(c + b_j + U(x_i))} \right)^{2.(a_j + w_j)} - (a_j + w_j)$$

$$\ln\left\{ \frac{b_j + U(x_i)}{c + b_j + U(x_i)} \right\} - c.\frac{a_j + w_j}{b_j + U(x_i)} - 1.$$

For the non-informative priors $a_1 = b_1 = a_2 = b_2 = 0$, the Bayes estimators coincide with the corresponding MLE's.

5. CONFIDENCE INTERVALS:

In this section, we propose two different confidence intervals. One is based on the asymptotic distribution of λ_1 and λ_2 and the second is the credible intervals based on the posterior distribution.

The $100(1-\alpha)\%$ approximate confidence intervals for λ_1 and λ_2 can be obtained using the asymptotic normality of the MLEs as follows

$$\hat{\lambda}_1 \pm z_{\alpha/2}\sqrt{\mathrm{Var}(\hat{\lambda}_1)}$$

and

$$\hat{\lambda}_2 \pm z_{\alpha/2}\sqrt{\mathrm{Var}(\hat{\lambda}_2)}$$

where $\mathrm{Var}(\hat{\lambda}_1) = \hat{\lambda}_1^2 / \mathrm{w}_1$, $\mathrm{Var}(\hat{\lambda}_2) = \hat{\lambda}_2^2 / \mathrm{w}_2$ and $z_{\alpha/2}$ is the upper $(\frac{\alpha}{2})^{th}$ percentile point of a standard normal distribution.

The $100(1-\alpha)\%$ two sides Bayes credible intervals of λ_1 and λ_2, say (l_j, u_j) can be obtained by solving the following two equations with respect to l_j and u_j

$$\int_0^{l_j} g(\lambda_j \mid \underline{x}).d\lambda_j = \alpha/2$$

and

$$\int_0^{u_j} g(\lambda_j \mid \underline{x}).d\lambda_j = 1 - \alpha/2, \; i = 1,2\,.$$

where l_j and u_j are the lower and upper limits of the interval. Using the posterior distributions of λ_1 and λ_2, the posterior of $Z_1 = 2\lambda_1(b_1 + U(x_i))$ and $Z_2 = 2\lambda_2 (b_2 + U(x_i))$ follows χ^2 distribution with $[2(a_1 + w_1)]$ and $[2(a_2 + w_2)]$ degrees of freedom respectively. Therefore, $100(1-\alpha)\%$ credible intervals for λ_1 and λ_2 are

$$\left\{ \frac{\chi^2_{[2(a_1+w_1)],1-\frac{\alpha}{2}}}{2(b_1 + U(x_i))}, \frac{\chi^2_{[2(a_1+w_1)],\frac{\alpha}{2}}}{2(b_1 + U(x_i))} \right\}$$

and

$$\left\{ \frac{\chi^2_{[2(a_2+w_2)],1-\frac{\alpha}{2}}}{2(b_2 + U(x_i))}, \frac{\chi^2_{[2(a_2+w_2)],\frac{\alpha}{2}}}{2(b_2 + U(x_i))} \right\}.$$

where $(a_1 + w_1) > 0$ and $(a_2 + w_2) > 0$. Note that if $[2(a_1 + w_1)]$ and $[2(a_2 + w_2)]$ are not integer values, then gamma distribution can be used to construct the credible intervals.

6. NUMERICAL RESULETS:

The performance of different results obtained in the previous sections can't be compared theoretical illustrative is very hard to obtain, , to illustrate the behavior of the proposed methods as well as evaluate the statistical performances of these estimates a numerical illustration is conducted. We re-analyze a real data set analyzed by Hoel (1972), and reused by Kundu et al. (2004). Also, a simulations study is used to compare the performance of the different estimators, different confidence intervals using different parameter values and different schemes.

We re-analyze one data set which was originally analyzed by Hoel (1972) and later by Kundu et al. (2004), Pareek et al. (2009) and Cramer and Schmiedt (2011) and Ashour and Nassar (2014). The data was obtained from a laboratory experiment in which male mice received a radiation dose of 300 roentgens at 35 days to 42 days (5-6 weeks) of age. The cause of death for each mouse was determined by reticulum cell sarcoma as cause 1 and other causes of death as cause 2, there were $n = 77$ observations remain in the analysis. The progressively Type-II censored data was generated and first used by Kundu et al. (2004). Considering $T = 700$ and using the censoring scheme $r = 20$ and $R_1 = ... = R_{20} = 2$, the hybrid progressive Type-II censored sample from the original data is given by
(40,2), (206,2), (282,2), (333,2), (341,2), (366,2), (407,2), (431,2), (462,2), (549,1), (558,1), (564,2), (571,1), (586,1),(586,2), (619,2), (620,2), (621,1), (631,2), (643,1), (661,1), (686,2), (697,1)
The first component denotes the life time and the second component indicate the cause of failure. There where $D = 23$, $D_1 = 8$, $r_2 = 15$ and $R_{21} = ... = R_{23} = 0$. From the above data, we obtain the following:

$$\sum_{i=1}^{r}(R_i + 1)x_i + \sum_{i=r+1}^{D} x_i + T\ R_D^* = 41116$$

which yields

Parameters	ML estimate	Bayes estimate under SE loss
λ_1	1.9457146×10^{-4}	2.189×10^{-4}
	4.732×10^{-9}	1.12974×10^{-7}
λ_2	3.6482148×10^{-4}	3.891×10^{-4}
	8.873×10^{-8}	9.4638×10^{-9}

where the variances and the Bayes risk reported within brackets. Also, the relative risk due to cause 1 is 0.348, and due to cause 2 is 0.652, The MLE's of the mean lifetimes due to cause 1 and cause 2 are given by $\hat{\lambda}_1^{-1} = 5139.5$ and $\hat{\lambda}_2^{-1} = 2741.067$. Now we report the 95% asymptotic, credible intervals and confidence intervals inTable 1.

The analysis of the previous real data set demonstrates the importance and usefulness of Type-II progressive hybrid censoring scheme and inferential procedures based on them.

A simulation study is conducted to evaluate the behavior of the ML and Bayes estimates by considering different values of sample sizes $n = 30, 50, 100$, different effective number of failures $r = 5, 10$ and $T = 0.4, 0.6$, and by choosing $(\lambda_1, \lambda_2) = (0.4, 0.6)$ and $(1, 0.8)$ in all the cases, and considered three different sampling schemes

Scheme 1: $R_1 = \cdots = R_{r-1} = 0$ and $R_r = n - r$,
Scheme 2: $R_1 = \cdots = R_{r-1} = 1$ *and* $R_r = n - 2r + 1$, and
Scheme 3: $R_1 = \cdots = R_{r-1} = R_r = (n - r)/r$.

The Bayes estimates are all computed by considering two types of priors, Prior 0: $a_1 = b_1 = a_2 = b_2 = 0$; Prior1: $a_1 = 1, b_1 = 1.5$, $a_2 = 1, b_2 = 1, c = 0.1$ It is noted that, Prior 0 describes the case of non-informative prior, whereas prior 1 has been selected arbitrarily. In each setting, we obtain the MLEs and Bayes estimates under SE and LINEX (with $c = 0.1$) loss functions of λ_1 and λ_2 based on 1000 simulations, with the assumption that the number of failures due to each cause of failures at least one, and the parameters distributed as a random variables with gamma prior distributions with parameter. The average values, average bias, root mean squared errors, estimates and average number of observed failures D_A for the ML and Bayes estimates of λ_1 and λ_2 are reported in Tables (2) and(3) . The average 95% confidence length of asymptotic confidence intervals and the credible intervals with respect to the gamma prior distributions, confidence intervals of λ_1 and λ_2 and the corresponding coverage probabilities are reported in Tables (4). All of the computations were performed using MATHCAD program version 2007 and are listed in the appendices.

From table (2) and (3), we observed that in most cases the MLE of λ_1 and λ_2 have smaller biases than the Bayes estimates and the Bayes estimates of λ_1 and λ_2 under SE loss function gives smaller biases and MSEs. The Bayes estimates based on prior 1 perform better than prior 0 in terms of minimum biases, MSEs and the lengths of the credible intervals. In most cases, the ordering of performance of the Bayes estimates of λ_1 and λ_2 in terms of minimum biases and MSEs (from best to worst) are Bayes estimates under SE loss function based on prior 1, Bayes estimates under LINEX loss function based on prior 1, Bayes estimates under SE loss function based on prior 0, Bayes estimates under LINEX loss function based on prior 0. As expected, it can be seen that the Bayes estimates based on the non-informative priors is quite -close to the MLEs. From Table (4), in terms of the coverage probabilities it can be seen that the approximate confidence intervals quite close to the nominal level than the Bayes credible intervals using prior 0 and prior 1, while, the Bayes credible intervals using prior 1 gives the smallest average confidence length.

Comparing the three censoring scheme based on minimum root mean squared errors shows that the performance of estimation for scheme 1 is best followed by scheme 2 and then scheme 3. When *T* becomes larger, the root mean square errors decreases, this is not being very surprising, because when *T* increases some additional information is gathered.

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Table (1)

Methods	λ_1	λ_2
Approximate	$(1.9456218\times10^{-4}, 1.9458073\times10^{-4})$	$(3.6480409\times10^{-4}, 3.6483887\times10^{-4})$
Bayes	$(1.000881\times10^{-4}, 3.834\times10^{-4})$	$(2.224207\times10^{-4}, 6.016956\times10^{-4})$

Table (2). The average biases , root mean squared errors and average number of failures of the ML and Bayes estimates of $(\lambda_1,\lambda_2)=(0.4,0.6)$ under different censoring scheme, different prior and different T 's.

Scheme	*(n ,r)*	ML Estimates		Bayes Estimates: SE Prior 0		SE Prior 1		LINEX Prior 0		LINEX Prior 1		D_A
		T = 0.4										
		$\hat{\lambda}_1$	$\hat{\lambda}_2$	$\tilde{\lambda}_{Sq1}$	$\tilde{\lambda}_{Sq2}$	$\tilde{\lambda}_{Sq1}$	$\tilde{\lambda}_{Sq2}$	$\tilde{\lambda}_{LIN1}$	$\tilde{\lambda}_{LIN2}$	$\tilde{\lambda}_{LIN1}$	$\tilde{\lambda}_{LIN2}$	
1	(30,5)	0.102	0.148	0.106	0.155	0.066	0.129	0.107	0.3072	0.068	0.267	9.891
		0.192	0.248	0.183	0.263	0.163	0.221	0.428	0.263	0.403	0.221	
2		0.093	0.14	0.099	0.144	0.054	0.121	0.1	0.3003	0.056	0.255	8.886
		0.191	0.253	0.196	0.251	0.159	0.221	0.442	0.251	0.398	0.221	
3		0.067	0.087	0.062	0.058	0.101	0.052	0.057	0.1432	0.097	0.103	5.375
		0.333	0.404	0.324	0.389	0.247	0.268	0.561	0.389	0.493	0.268	
1	(50,5)	0.081	0.113	0.08	0.128	0.059	0.102	0.081	0.2813	0.06	0.259	16.45
		0.161	0.21	0.158	0.212	0.143	0.194	0.398	0.212	0.378	0.194	
2		0.071	0.113	0.077	0.127	0.048	0.101	0.078	0.2778	0.048	0.248	15.52
		0.159	0.21	0.16	0.216	0.141	0.193	0.4	0.216	0.048	0.193	
3		0.001	0.031	0.017	0.04	0.039	0.03	0.02	0.2202	0.036	0.163	6.444
		0.258	0.302	0.215	0.277	0.206	0.228	0.462	0.277	0.451	0.228	
1	(100,5)	0.050	0.069	0.05	0.069	0.038	0.064	0.051	0.2505	0.039	0.238	32.88
		0.113	0.147	0.112	0.144	0.105	0.141	0.335	0.144	0.325	0.141	
2		0.047	0.075	0.048	0.074	0.525	0.069	0.048	0.2482	0.035	0.235	31.61
		0.116	0.158	0.115	0.156	0.108	0.151	0.339	0.156	0.328	0.151	
3		0.054	0.08	0.045	0.079	0.019	0.067	0.046	0.2463	0.021	0.220	9.608
		0.19	0.243	0.195	0.246	0.165	0.213	0.441	0.246	0.405	0.213	
	(30,10)	0.043	0.085	0.05	0.074	0.012	0.071	0.051	0.251	0.013	0.213	10.90
		0.157	0.192	0.164	0.192	0.137	0.169	0.404	0.192	0.369	0.169	
2		0.011	0.013	0.018	0.017	0.041	0.015	0.016	0.184	0.039	0.160	10.18
		0.199	0.229	0.197	0.238	0.179	0.196	0.441	0.238	0.421	0.196	
3		0.033	0.054	0.043	0.077	0.061	0.048	0.04	0.1595	0.058	0.141	10.00
		0.223	0.275	0.236	0.313	0.199	0.229	0.483	0.313	0.444	0.229	
1	(50,10)	0.125	0.213	0.126	0.183	0.103	0.171	0.126	0.3262	0.104	0.303	16.57
		0.17	0.185	0.169	0.241	0.15	0.222	0.411	0.241	0.388	0.222	
2		0.118	0.178	0.116	0.169	0.093	0.162	0.117	0.3172	0.094	0.293	14.25
		0.168	0.231	0.174	0.229	0.146	0.213	0.417	0.229	0.382	0.213	
3		0.037	0.04	0.042	0.064	0.064	0.036	0.039	0.1608	0.062	0.138	10.07
		0.218	0.259	0.234	0.307	0.195	0.218	0.481	0.307	0.439	0.218	
1	(100,10)	0.084	0.12	0.083	0.127	0.073	0.114	0.084	0.2837	0.073	0.273	33.11
		0.126	0.169	0.125	0.178	0.117	0.162	0.354	0.178	0.342	0.162	
2		0.087	0.124	0.088	0.132	0.074	0.118	0.089	0.2887	0.075	0.274	30.36
		0.131	0.177	0.131	0.18	0.12	0.169	0.362	0.18	0.347	0.169	

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3		0.011	0.024	0.006	0.003	0.018	0.017	0.008	0.2077	0.017	0.183	11.05
		0.176	0.21	0.188	0.225	0.157	0.184	0.432	0.225	0.395	0.184	
						***T* = 0.6**						
1	(30,5)	0.080	0.132	0.082	0.13	0.054	0.118	0.083	0.2834	0.055	0.255	13.51
		0.166	0.233	0.171	0.23	0.146	0.213	0.413	0.23	0.382	0.213	
2		0.093	0.123	0.088	0.132	0.063	0.109	0.09	0.2895	0.064	0.264	11.94
		0.176	0.238	0.181	0.241	0.151	0.216	0.425	0.241	0.389	0.216	
3		0.003	0.008	0.01	0.012	0.047	0	0.007	0.1934	0.044	0.156	6.033
		0.246	0.34	0.254	0.311	0.2	0.245	0.501	0.311	0.444	0.245	
1	(50,5)	0.067	0.093	0.058	0.098	0.05	0.085	0.058	0.2582	0.051	0.250	22.36
		0.136	0.182	0.137	0.18	0.124	0.171	0.37	0.18	0.352	0.171	
2		0.066	0.088	0.068	0.088	0.048	0.08	0.068	0.2682	0.048	0.248	20.9
		0.141	0.181	0.142	0.183	0.128	0.169	0.377	0.183	0.358	0.169	
3		0.051	0.068	0.05	0.07	0.009	0.058	0.052	0.2522	0.011	0.210	7.69
		0.205	0.272	0.21	0.278	0.172	0.226	0.457	0.278	0.414	0.226	
1	(100,5)	0.036	0.05	0.037	0.061	0.028	0.046	0.037	0.2372	0.028	0.228	45.22
		0.097	0.124	0.097	0.124	0.092	0.12	0.312	0.124	0.303	0.12	
2		0.040	0.061	0.039	0.057	0.031	0.057	0.039	0.2389	0.032	0.231	42.96
		0.098	0.127	0.097	0.129	0.092	0.122	0.311	0.129	0.304	0.122	
3		0.045	0.067	0.042	0.068	0.016	0.057	0.043	0.2432	0.018	0.217	12.30
		0.173	0.218	0.175	0.216	0.153	0.195	0.417	0.216	0.391	0.195	
1	(30,10)	0.121	0.17	0.114	0.173	0.094	0.154	0.115	0.3152	0.095	0.295	13.61
		0.121	0.227	0.17	0.227	0.146	0.208	0.412	0.227	0.382	0.208	
2		0.054	0.079	0.053	0.07	0.023	0.066	0.054	0.2542	0.024	0.223	11.22
		0.162	0.194	0.16	0.209	0.14	0.171	0.399	0.209	0.374	0.171	
3		0.044	0.061	0.04	0.073	0.069	0.052	0.037	0.1626	0.067	0.133	10.03
		0.241	0.299	0.226	0.303	0.213	0.247	0.473	0.303	0.46	0.247	
1	(50,10)	0.101	0.153	0.103	0.155	0.085	0.143	0.104	0.304	0.085	0.285	22.70
		0.149	0.205	0.147	0.207	0.135	0.194	0.384	0.207	0.367	0.194	
2		0.099	0.159	0.103	0.158	0.081	0.147	0.103	0.3035	0.081	0.281	19.18
		0.15	0.214	0.153	0.217	0.134	0.201	0.391	0.217	0.367	0.201	
3		0.019	0.036	0.021	0.045	0.047	0.033	0.019	0.1809	0.045	0.154	10.39
		0.208	0.258	0.209	0.27	0.185	0.218	0.455	0.27	0.428	0.218	
1	(100,10)	0.062	0.095	0.06	0.093	0.054	0.09	0.061	0.2607	0.054	0.254	45.28
		0.106	0.146	0.106	0.143	0.1	0.141	0.325	0.143	0.316	0.141	
2		0.062	0.1	0.061	0.092	0.053	0.095	0.061	0.2615	0.053	0.253	41.43
		0.107	0.148	0.107	0.146	0.1	0.143	0.328	0.146	0.316	0.143	
3		0.035	0.049	0.036	0.054	0.007	0.041	0.037	0.2371	0.008	0.208	12.38
		0.16	0.223	0.173	0.204	0.142	0.199	0.415	0.204	0.376	0.199	

The first and the second row in each cell represent the average biases and root mean squared errors of λ_1 and λ_2 respectively.

Table (3). The average biases, root mean squared errors and average number of failures of the ML and Bayes estimates of $(\lambda_1, \lambda_2) = (1, 0.8)$ under different censoring scheme, different prior and different *T* 's.

Scheme	*(n ,r)*	ML Estimates		Bayes Estimates								D_A
				SE				LINEX				
				Prior 0		Prior 1		Prior 0		Prior 1		
						***T* = 0.4**						
		$\hat{\lambda}_1$	$\hat{\lambda}_2$	$\tilde{\lambda}_{Sq1}$	$\tilde{\lambda}_{Sq2}$	$\tilde{\lambda}_{Sq1}$	$\tilde{\lambda}_{Sq2}$	$\tilde{\lambda}_{LIN1}$	$\tilde{\lambda}_{LIN2}$	$\tilde{\lambda}_{LIN1}$	$\tilde{\lambda}_{LIN2}$	
1	(30,5)	0.193	0.15	0.199	0.162	0.214	0.151	0.203	0.0028	0.217	0.016	15.40
		0.362	0.314	0.361	0.309	0.341	0.284	0.601	0.309	0.584	0.284	
2		0.179	0.165	0.203	0.149	0.205	0.164	0.207	0.0073	0.208	0.008	13.54
		0.373	0.333	0.369	0.318	0.345	0.298	0.608	0.318	0.588	0.298	

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3		0.033	0.013	0.02	0.04	0.139	0.072	0.035	0.1654	0.147	0.052	6.394
		0.516	0.506	0.542	0.427	0.359	0.321	0.727	0.427	0.598	0.321	
1	(50,5)	0.148	0.103	0.147	0.115	0.165	0.106	0.15	0.0505	0.168	0.032	25.54
		0.295	0.237	0.288	0.254	0.285	0.222	0.537	0.254	0.534	0.222	
2		0.139	0.113	0.137	0.109	0.159	0.116	0.14	0.0598	0.161	0.039	23.67
		0.288	0.249	0.284	0.257	0.278	0.232	0.533	0.257	0.528	0.232	
3		0.115	0.09	0.125	0.096	0.173	0.109	0.133	0.0667	0.179	0.021	8.483
		0.425	0.389	0.434	0.395	0.353	0.306	0.657	0.395	0.595	0.306	
1	(100,5)	0.074	0.057	0.073	0.061	0.086	0.061	0.074	0.1257	0.088	0.112	51.50
		0.204	0.176	0.19	0.172	0.2	0.17	0.436	0.172	0.447	0.17	
2		0.076	0.063	0.08	0.057	0.09	0.066	0.081	0.1187	0.091	0.109	49.38
		0.196	0.178	0.202	0.171	0.193	0.172	0.45	0.171	0.44	0.172	
3		0.102	0.075	0.12	0.063	0.142	0.088	0.125	0.0747	0.147	0.053	13.55
		0.363	0.314	0.351	0.318	0.322	0.27	0.592	0.318	0.567	0.27	
	(30,10)	0.298	0.236	0.301	0.233	0.305	0.227	0.304	0.1038	0.308	0.107	15.41
		0.396	0.335	0.397	0.322	0.382	0.31	0.631	0.322	0.619	0.31	
2		0.187	0.165	0.191	0.152	0.214	0.164	0.196	0.0041	0.218	0.017	12.06
		0.356	0.314	0.351	0.307	0.33	0.277	0.593	0.307	0.575	0.277	
3		0.086	0.08	0.114	0.083	0.026	0.017	0.102	0.3023	0.034	0.166	10.08
		0.496	0.43	0.491	0.468	0.345	0.308	0.693	0.468	0.584	0.308	
1	(50,10)	0.233	0.178	0.227	0.183	0.242	0.175	0.229	0.0293	0.244	0.044	25.83
		0.323	0.265	0.312	0.275	0.317	0.253	0.559	0.275	0.564	0.253	
2		0.232	0.197	0.237	0.186	0.243	0.193	0.239	0.0394	0.245	0.045	21.59
		0.333	0.287	0.337	0.285	0.326	0.271	0.582	0.285	0.572	0.271	
3		0.020	0.012	0.035	0.03	0.063	0.028	0.026	0.2257	0.07	0.13	10.62
		0.402	0.375	0.415	0.392	0.301	0.286	0.639	0.392	0.548	0.286	
1	(100,10)	0.144	0.115	0.141	0.115	0.153	0.116	0.142	0.058	0.154	0.046	51.30
		0.222	0.191	0.215	0.193	0.222	0.187	0.465	0.193	0.472	0.187	
2		0.149	0.106	0.151	0.115	0.159	0.108	0.153	0.0471	0.16	0.040	47.15
		0.236	0.194	0.234	0.195	0.235	0.188	0.484	0.195	0.485	0.188	
3		0.095	0.096	0.1	0.073	0.139	0.107	0.106	0.094	0.143	0.056	13.09
		0.35	0.316	0.369	0.313	0.307	0.27	0.607	0.313	0.554	0.27	
						***T* = 0.6**						
1	(30,5)	0.157	0.121	0.154	0.12	0.178	0.124	0.158	0.0423	0.181	0.018	19.77
		0.328	0.273	0.317	0.278	0.311	0.251	0.564	0.278	0.558	0.251	
2		0.140	0.127	0.15	0.11	0.167	0.131	0.154	0.0459	0.17	0.03	17.37
		0.341	0.295	0.339	0.284	0.317	0.267	0.582	0.284	0.563	0.267	
3		0.084	0.059	0.073	0.074	0.165	0.097	0.084	0.1156	0.172	0.028	7.406
		0.503	0.445	0.477	0.433	0.371	0.325	0.686	0.433	0.608	0.325	
1	(50,5)	0.115	0.092	0.1	0.087	0.131	0.095	0.102	0.0976	0.133	0.067	32.91
		0.244	0.214	0.244	0.213	0.238	0.203	0.494	0.213	0.489	0.203	
2		0.124	0.088	0.112	0.09	0.141	0.092	0.114	0.086	0.143	0.057	30.34
		0.254	0.227	0.248	0.22	0.248	0.214	0.498	0.22	0.498	0.214	
3		0.106	0.074	0.094	0.09	0.16	0.096	0.102	0.098	0.165	0.035	10.10
		0.415	0.394	0.438	0.374	0.351	0.317	0.659	0.374	0.592	0.317	
1	(100,5)	0.056	0.049	0.06	0.05	0.067	0.052	0.061	0.1393	0.068	0.132	66.11
		0.171	0.15	0.17	0.155	0.169	0.146	0.412	0.155	0.412	0.146	
2		0.062	0.043	0.055	0.05	0.073	0.047	0.056	0.1442	0.074	0.125	63.54
		0.174	0.156	0.172	0.159	0.172	0.151	0.415	0.159	0.414	0.151	
3		0.073	0.05	0.083	0.07	0.111	0.065	0.088	0.1125	0.115	0.085	16.78
		0.329	0.299	0.318	0.283	0.294	0.261	0.563	0.283	0.542	0.261	
1	(30,10)	0.268	0.209	0.259	0.215	0.276	0.204	0.262	0.0615	0.278	0.078	19.65
		0.36	0.298	0.361	0.3	0.351	0.28	0.602	0.3	0.593	0.28	
2		0.239	0.174	0.243	0.18	0.255	0.173	0.247	0.0468	0.258	0.057	14.60
		0.369	0.313	0.378	0.325	0.351	0.284	0.615	0.325	0.594	0.284	

3		0.077	0.044	0.058	0.041	0.026	0.006	0.047	0.2475	0.033	0.166	10.38
		0.464	0.397	0.468	0.418	0.329	0.292	0.678	0.418	0.571	0.292	
1	(50,10)	0.178	0.157	0.19	0.158	0.189	0.156	0.192	0.0083	0.191	0.009	33.14
		0.272	0.241	0.283	0.239	0.269	0.232	0.533	0.239	0.52	0.232	
2		0.198	0.152	0.197	0.156	0.21	0.152	0.2	0.0004	0.212	0.011	27.43
		0.293	0.254	0.297	0.255	0.289	0.241	0.546	0.255	0.538	0.241	
3		0.027	0.069	0.05	0.047	0.092	0.087	0.058	0.142	0.098	0.102	11.47
		0.374	0.33	0.401	0.337	0.305	0.271	0.63	0.337	0.551	0.271	
1	(100,10)	0.117	0.088	0.12	0.093	0.126	0.089	0.121	0.0789	0.127	0.073	66.07
		0.194	0.161	0.195	0.169	0.195	0.158	0.442	0.169	0.442	0.158	
2		0.114	0.087	0.124	0.097	0.123	0.089	0.126	0.0744	0.124	0.075	60.51
		0.198	0.172	0.206	0.168	0.198	0.168	0.454	0.168	0.446	0.168	
3		0.092	0.077	0.11	0.074	0.132	0.09	0.115	0.085	0.136	0.064	14.88
		0.345	0.304	0.348	0.308	0.306	0.263	0.589	0.308	0.553	0.263	

The first and the second row in each cell represent the average biases and root mean squared errors of λ_1 and λ_2 respectively.

Table (4). The average 95% confidence lengths and the coverage probabilities $(\lambda_1, \lambda_2) = (0.4, 0.6)$ and $(\lambda_1, \lambda_2) = (1, 0.8)$ for different methods and different censoring scheme, different prior and different *T* 's.

Shceme	*(n,r)*	*Parameters*	*Prior*	*Approximate*	*Bayes*	*Approximate*	*Bayes*
				T = 0.4			
				$\lambda_1 = 0.4$ and $\lambda_2 = 0.6$		$\lambda_1 = 1$ and $\lambda_2 = 0.8$	
1		λ_1	Prior 0	—	0.55 (82.5)	—	1.044 (83.3)
			Prior 1	0.721 (95)	0.565 (90.1)	0.904 (67.7)	0.974 (81.7)
		λ_2	Prior 0	—	0.686 (77.1)	—	0.925 (82.5)
			Prior 1	0.861 (72.9)	0.678 (83.9)	0.792 (73.4)	0.881 (84.8)
2	(30,5)	λ_1	Prior 0	—	0.592(81.9)	—	1.107 (84.7)
			Prior 1	0.77 (97.5)	0.61 (93.2)	0.953 (68.1)	1.032 (83.2)
		λ_2	Prior 0	—	0.745 (82.3)	—	0.993 (85.2)
			Prior 1	0.916 (76.1)	0.726 (87)	0.81 (71.2)	0.917 (84.9)
3		λ_1	Prior 0	—	1.136 (97.5)	—	1.93 (92.8)
			Prior 1	1.418 (99.9)	1.053 (99.1)	1.791 (94.5)	1.533 (95.5)
		λ_2	Prior 0	—	1.384 (94.4)	—	1.676 (94.5)
			Prior 1	1.689 (91)	1.224 (98.4)	1.585 (82.4)	1.406 (97.8)
1		λ_1	Prior 0	—	0.47 (82.9)	—	0.874 (84.3)
			Prior 1	0.645 (96.4)	0.473 (88.2)	0.726 (69)	0.83 (84.1)
		λ_2	Prior 0	—	0.578 (79.7)	—	0.78 (85.8)
			Prior 1	0.762 (77.1)	0.576 (85.3)	0.642 (78.1)	0.755 (88.8)
2	(50,5)	λ_1	Prior 0	—	0.489 (83.8)	—	0.913 (87)
			Prior 1	0.66 (96.9)	0.497 (90.6)	0.746 (70.1)	0.863 (85.8)
		λ_2	Prior 0	—	0.599 (79.7)	—	0.813 (86.2)
			Prior 1	0.774 (79.3)	0.596 (86.1)	0.646 (77.4)	0.775 (89.1)
3		λ_1	Prior 0	—	0.873 (94.9)	—	1.512 (89.2)
			Prior 1	1.097 (99.9)	0.863 (98.5)	0.792 (71.7)	0.9 (87.8)
		λ_2	Prior 0	—	1.079 (93.3)	—	1.343 (88)
			Prior 1	1.29 (89.6)	0.999 (97.4)	0.687 (77.8)	0.809 (87.7)
		λ_1	Prior 0	—	0.373 (89)	—	0.673 (91.9)

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			Prior 1	0.556 (99.3)	0.372 (90.9)	0.492 (64.6)	0.653 (88.5)
1		λ_2	Prior 0	—	0.462 (88.6)	—	0.6 (91.1)
			Prior 1	0.645 (83.8)	0.455 (88)	0.421 (74.1)	0.586 (90.7)
		λ_1	Prior 0	—	0.38 (88.2)	—	0.683 (89.3)
	(100,5)		Prior 1	0.556 (99.1)	0.38 (91.3)	0.501 (65.5)	0.664 (89.7)
2		λ_2	Prior 0	—	0.467 (85.5)	—	0.613 (91.4)
			Prior 1	0.642 (82.1)	0.46 (86.5)	0.427 (73.8)	0.595 (90.9)
		λ_1	Prior 0	—	0.665 (87.3)	—	1.23 (89.5)
			Prior 1	0.841 (98.1)	0.651 (94.7)	0.512 (65.7)	0.673 (89.8)
3		λ_2	Prior 0	—	0.822 (86.5)	—	1.118 (91.3)
			Prior 1	1.002 (81.9)	0.78 (91)	0.437 (77.1)	0.603 (91.2)
		λ_1	Prior 0	—	0.631 (92.9)	—	0.913 (70.9)
			Prior 1	0.811 (100)	0.635 (97.6)	0.788 (48.2)	0.862 (70.3)
1		λ_2	Prior 0	—	0.787 (92.8)	—	0.818 (76.9)
			Prior 1	0.949 (89.5)	0.748 (96.2)	0.688 (65.7)	0.779 (76.7)
		λ_1	Prior 0	—	0.773 (94.6)	—	1.191 (89)
2	(30,10)		Prior 1	0.982 (100)	0.744 (97.2)	1.032 (75.9)	1.083 (88.4)
		λ_2	Prior 0	—	0.957 (95)	—	1.057 (89)
			Prior 1	1.166 (93.2)	0.888 (96.2)	0.885 (78.1)	0.969 (90.7)
		λ_1	Prior 0	—	0.837 (93.7)	—	1.79 (94.9)
3			Prior 1	1.055 (100)	0.785 (97)	1.576 (95)	1.457 (94.5)
		λ_2	Prior 0	—	1.054 (94.1)	—	1.574 (92.8)
			Prior 1	1.257 (93.9)	0.941 (96.8)	1.387 (82.8)	1.328 (97.8)
		λ_1	Prior 0	—	0.405 (74.1)	—	0.782 (72)
1			Prior 1	0.549 (92.6)	0.41 (79)	0.643 (50.7)	0.748 (72)
		λ_2	Prior 0	—	0.504 (64.4)	—	0.698 (76.7)
			Prior 1	0.647 (59)	0.496 (68.9)	0.564 (69.4)	0.678 (78.4)
		λ_1	Prior 0	—	0.448 (79.7)	—	0.851 (76.6)
2	(50,10)		Prior 1	0.592 (96.8)	0.452 (87.2)	0.689 (51.6)	0.806 (76.4)
		λ_2	Prior 0	—	0.561 (75.1)	—	0.753 (79.4)
			Prior 1	0.699 (66.3)	0.545 (77.1)	0.59 (68.8)	0.719 (79.6)
		λ_1	Prior 0	—	0.826 (93.1)	—	1.624 (95.7)
			Prior 1	1.035 (99.8)	0.784 (97.7)	1.418 (94.6)	1.367 (96.7)
3		λ_2	Prior 0	—	1.032 (94.2)	—	1.446 (93.5)
			Prior 1	1.218 (93.9)	0.927 (96.8)	1.234 (84.7)	1.234 (95.6)
		λ_1	Prior 0	—	0.335 (79.7)	—	0.625 (82.6)
1			Prior 1	0.502 (98.3)	0.335 (82.4)	0.455 (47.7)	0.607 (81.1)
		λ_2	Prior 0	—	0.412 (71.4)	—	0.556 (83.6)
			Prior 1	0.582 (69.7)	0.411 (77.1)	0.388 (74.2)	0.544 (84.4)
		λ_1	Prior 0	—	0.345 (79.3)	—	0.647 (80.4)
			Prior 1	0.511 (98.3)	0.347 (81.3)	0.494 (52.8)	0.629 (80.4)
2	(100,10)	λ_2	Prior 0	—	0.426 (72.8)	—	0.58 (85.5)
			Prior 1	0.594 (69.9)	0.425 (76.9)	0.43 (75.5)	0.57 (85.4)
		λ_1	Prior 0	—	0.707 (92.9)	—	1.275 (89.9)
			Prior 1	0.94 (99.8)	0.704 (96.3)	1.089 (84.3)	1.144 (91.9)
3		λ_2	Prior 0	—	0.885 (94.9)	—	1.139 (92.2)
			Prior 1	1.118 (94.2)	0.848 (97.5)	0.931 (80.4)	1.02 (92.7)
				T = 0.6			
				$\lambda_1 = 0.4$ and $\lambda_2 = 0.6$		$\lambda_1 = 1$ and $\lambda_2 = 0.8$	
		λ_1	Prior 0	—	0.508 (82.5)	—	0.979 (84.4)
			Prior 1	0.668 (94.2)	0.514 (89.8)	0.82(69.3)	0.914 (83.1)
1		λ_2	Prior 0	—	0.626 (77.5)	—	0.873 (84.8)

			Prior 1	0.781 (74.6)	0.611 (81.2)	0.717 (75.3)	0.826 (87.9)
		λ_1	Prior 0		0.531 (82.8)	—	1.048 (85.8)
	(30,5)		Prior 1	0.708 (95.3)	0.533 (88.1)	0.876 (71.9)	0.976 (85.2)
2		λ_2	Prior 0	—	0.662 (78.3)	—	0.94 (87.2)
			Prior 1	0.847 (75.6)	0.651 (83.4)	0.751 (75)	0.871 (87.5)
		λ_1	Prior 0	—	0.96 (95.8)	—	1.69 (90.7)
			Prior 1	1.19 (99.6)	0.909 (99.4)	1.555 (86.4)	1.404 (91.6)
3		λ_2	Prior 0	—	1.175 (93.4)	—	1.477 (90.9)
			Prior 1	1.429 (90.2)	1.07 (97.4)	1.367 (79.2)	1.282 (94.4)
		λ_1	Prior 0	—	0.432 (86.7)	—	0.81 (89.5)
			Prior 1	0.604 (97.8)	0.427 (89.9)	0.628 (66.1)	0.767 (87.6)
1		λ_2	Prior 0	—	0.527 (82.4)	—	0.719 (88.7)
			Prior 1	0.708 (80)	0.521 (86.3)	0.542 (78.8)	0.689 (90.7)
		λ_1	Prior 0	—	0.441 (85.4)	—	0.837 (88.7)
			Prior 1	0.626 (97.9)	0.443 (89.7)	0.669 (68)	0.79 (87.4)
2		λ_2	Prior 0	—	0.553 (83.5)	—	0.746 (89.9)
	(50,5)		Prior 1	0.735 (82.7)	0.542 (87.4)	0.586 (78.4)	0.715 (89.2)
		λ_1	Prior 0	—	0.736 (90.2)	—	1.435 (90.1)
			Prior 1	0.938 (98.8)	0.73 (96.9)	1.28 (82.7)	1.246 (90.2)
3		λ_2	Prior 0	—	0.925 (88.2)	—	1.255 (88.4)
			Prior 1	1.128 (83.9)	0.872 (93.6)	1.127 (77.9)	1.134 (90)
		λ_1	Prior 0	—	0.331 (89.5)	—	0.605 (91.8)
1			Prior 1	0.52 (99.5)	0.331 (91.4)	0.425 (63.1)	75.5 (92.2)
		λ_2	Prior 0	—	0.405 (89.6)	—	0.539 (91.6)
		λ_2	Prior 1	0.598 (86.7)	0.405 (89.5)	0.363 (75.5)	0.531 (92.2)
		λ_1	Prior 0	—	0.337 (90.4)	—	0.619 (90.8)
2	(100,5)		Prior 1	0.517 (99.4)	0.335 (91.8)	0.437 (62.3)	0.6 (90.7)
		λ_2	Prior 0	—	0.414 (87.7)	—	0.55 (91.1)
			Prior 1	0.593 (85.7)	0.408 (88.4)	0.374 (75.6)	0.541 (92.3)
		λ_1	Prior 0	—	0.602 (90.3)	—	1.153 (90.5)
			Prior 1	0.788 (99)	0.597 (93.4)	0.999 (81.5)	1.065 (90.3)
3		λ_2	Prior 0	—	0.745 (88.4)	—	1.022 (91)
			Prior 1	0.932 (85.9)	0.718 (91.9)	0.876 (79.5)	0.964 (91.2)
		λ_1	Prior 0	—	0.461 (81.3)	—	0.855 (72.8)
1			Prior 1	0.615 (97.9)	0.463 (88.4)	0.718 (49.7)	0.808 (72.5)
		λ_2	Prior 0	—	0.572 (76.9)	—	0.756 (75)
			Prior 1	0.732 (69.2)	0.564 (81)	0.628 (68.7)	0.731 (78.7)
		λ_1	Prior 0	—	0.621 (93.5)	—	1.016 (78.7)
2			Prior 1	0.804 (99.9)	0.614 (96.8)	0.902 (63.9)	0.952 (78.7)
	(30,10)	λ_2	Prior 0	—	0.781 (93.6)	—	0.913 (81.4)
			Prior 1	0.956 (90.7)	0.739 (96.7)	0.802 (74.9)	0.869 (85.1)
		λ_1	Prior 0	—	0.83 (95)	—	1.679 (94.7)
			Prior 1	1.065 (99.6)	0.795 (97)	1.497 (94.5)	1.431 (96.5)
3		λ_2	Prior 0	—	1.047 (93.9)	—	1.481 (94.6)
			Prior 1	1.264 (93.5)	0.948 (96.2)	1.288 (81.4)	1.284 (96.5)
		λ_1	Prior 0	—	0.376 (77.6)	—	0.73 (77.1)
1			Prior 1	0.53 (95.4)	0.38 (81.9)	0.563 (50)	0.71 (79.2)
		λ_2	Prior 0	—	0.465 (69.7)	—	0.648 (79.6)
			Prior 1	0.618 (65.2)	0.461 (72.1)	0.476 (71.2)	0.631 (80)
		λ_1	Prior 0	—	0.408 (78.2)	—	0.793 (79.6)
2	(50,10)		Prior 1	0.555 (88.7)	0.412 (84.3)	0.646 (54.9)	0.76 (78.2)
		λ_2	Prior 0	—	0.502 (72.6)	—	0.707 (81.1)
			Prior 1	0.646 (66.8)	0.494 (74.5)	0.564 (73.1)	0.686 (81.6)

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		λ_1	Prior 0	——	0.781 (95.2)	——	1.43 (92.3)
			Prior 1	1.009 (99.8)	0.751 (96.8)	1.232 (89.8)	1.27 (94.4)
3		λ_2	Prior 0	——	0.986 (95)	——	1.262 (93.2)
			Prior 1	1.202 (93)	0.904 (96.5)	1.026(81.5)	1.117 (95.5)
		λ_1	Prior 0	——	0.308 (83.9)	——	0.566 (84.4)
1			Prior 1	0.464 (99)	0.305 (84.5)	0.398 (47.3)	0.558 (84)
		λ_2	Prior 0	——	0.378 (78.8)	——	0.507 (85.6)
			Prior 1	0.53 (72)	0.368 (77.4)	0.338 (73.9)	0.501 (86.3)
		λ_1	Prior 0	——	0.32 (84.7)	——	0.59 (85.3)
			Prior 1	0.484 (98.5)	0.315 (84)	0.421 (51.4)	0.581 (83.5)
2	(100,10)	λ_2	Prior 0	——	0.394 (80.2)	——	0.528 (87.5)
			Prior 1	0.557 (73.7)	0.384 (78.4)	0.358 (73.4)	0.521 (87.1)
		λ_1	Prior 0	——	0.619 (92.1)	——	1.189 (89.1)
			Prior 1	0.818 (99.7)	0.614 (96.4)	1.035(79.6)	1.095 (91)
3		λ_2	Prior 0	——	0.771 (92.6)	——	1.067 (90.3)
			Prior 1	0.969 (87.3)	0.74 (92.3)	0.899 (79.8)	0.985 (91.8)

The first and second rows represent the average 95% confidence lengths of asymptotic confidence intervals, the credible intervals with respect to the gamma prior distributions, Boot-p and Boot-t confidence intervals of λ_1 and λ_2 respectively, and the corresponding coverage probabilities are reported within brackets.