



MODELING AND FORECASTING DISPLACEMENTS IN CONCRETE DAM: AN ARDL BOUNDS TESTING APPROACH

¹SUELLEN RIBEIRO PARDO GARCIA, ²ANSELMO CHAVES NETO, ³SHEILA REGINA ORO, ⁴CLAUDIO NEUMANN JUNIOR

^{1,3}Department of Mathematics, Federal University of Technological - Paraná, Brazil

²Department of Mathematic and Statistic, Federal University of Paraná, Curitiba - PR, Brazil

⁴Civil Engineering and Architecture Division, Itaipu, Foz do Iguaçu, Brazil.

E-mail: ¹suellenpardo@utfpr.edu.br, ²anselmo@ufpr.br, ³sheilaro@utfpr.edu.br,
⁴neumann@itaipu.gov.br

ABSTRACT

In this paper is carried out modeling and forecasting of horizontal displacement, in the direction of flow, of a concrete block of Itaipu Hydroelectric Power Plant based on measurements of the direct pendulum and considering the influence of temperature variation and variation reservoir level. The methodology involves the Autoregressive Distributed Lags Model (ARDL) and the Bounds Testing approach Pesaran et al. (2001), which has some advantages over conventional cointegration tests when the time series is stationary (I(0)) and/or integrated of order one (I(1)). The main objective is to present a model for prediction of displacements providing a contribution to the technicians and engineers in decision-making with respect to the monitoring of this block. The Eviews software was used to estimate the model parameters. The analysis confirms the existence of a long-run relationship between the variables. It evaluated correct model specification, independence, normality and homoscedasticity of the residuals and, finally, the stability of the model, as well, the model could be used for prediction purposes. Accuracy measurements RMSE and MAPE were calculated and confidence intervals were built.

Keywords: *Displacement; Concrete Dam; ARDL; Bounds Testing; Direct Pendulum*

1. INTRODUCTION

Statistical models have been widely used to predict the response of a dam monitoring instrument. Such models are designed to detect changes in the dam's behavior in advance allowing the implementation of appropriate corrective measures helping to dam safety.

The models are based on correlations between factors such as the water level of the reservoir, ambient temperature, wear due to time and the dam's response to these actions as strain, creep and displacements [1]. Various applications are found in the literature, for example, [1] – [6].

There are several challenges to propose such models. One is that the independent variables

(variations of the reservoir level and temperature) can generate multicollinear data, so you cannot use some classical statistical techniques. Another challenge is the autocorrelation of the data because data are typically collected over time, that is, there is a possibility of a temporal autocorrelation. Finally, the time series can be integrated in different orders and eventually cointegrated.

In trying to overcome these challenges we propose the dynamic model ARDL whose regressors include lagged values of the dependent variable and current and lagged values of the independent variables. In fact, use the ARDL model in dam monitoring data is consistent, because the relationship between environmental variables and the response of the dam is not instantaneous. For example, the thermal inertia

creates a delay in response between the temperature variation and instrument readings.

The objective of the paper is to present an ARDL model for prediction purposes of horizontal displacements in the direction of flow of a concrete block of Itaipu Hydroelectric Power Plant based on measurements of the direct pendulum and considering the influence of temperature variation measured by surface thermometers and influence of the reservoir level variation. After validating the model, predictions are calculated and are built confidence intervals.

The importance of work is the methodology applied to dam monitoring data. The ARDL Bounds Testing approach is widespread as econometric model and none of the consulted references mentions similar to the application presented here. In literature often the statistical models dam monitoring data ignore the presence of multicollinearity and autocorrelation of the variables that are considered in this approach.

With valid model, is expected to contribute with technicians and engineers in decision-making with regard to the monitoring of this block. In fact, since it builds confidence interval for the displacements, can evaluate new series of readings over a possible abnormality and thus initiating an investigative process. The detection of a possible abnormality in the structure, at some instrument or the environment, if previously identified, corrective measures are taken in time to prevent further damage.

2. AUTOREGRESSIVE DISTRIBUTED LAGS MODEL

The ARDL model encompasses a class of dynamic models in which the regressors include lagged values of the dependent variable and current and lagged values of the independent variables. Follows a ARDL(r,s) model, where for simplicity we consider only one independent variable, x_t ,

$$y_t = \mu + \sum_{i=1}^r \alpha_i y_{t-i} + \sum_{i=0}^s \beta_i x_{t-i} + \varepsilon_t, \quad \varepsilon_t \sim iid(0, \sigma^2) \quad (1)$$

In (Eq. 1) μ is the independent term, the r and s indexes represent the number of maximum lag for the dependent variable, y_t , and independent x_t , respectively. The errors must be independent and identically distributed.

In the model specification, two aspects are crucial: the determination of lag orders (r and s) and the estimation of coefficients. In determining the lag order some alternatives include the choice of model that maximizes the coefficient of determination or minimizes the estimated variance of the errors, and the information criteria such as Akaike (AIC) and Bayesian Schwarz statistics (BIC). As for the estimation, linear transformations are applied in ARDL model and uses the method of estimation by ordinary least squares, which besides being consistent is invariant to linear transformations (ARONE, 2014).

2.1 The Error Correction Model (ECM)

The error correction model (ECM) is a parameter of an ARDL model and offers advantages such as alleviating common problems in the original ARDL model.

Using the delay operator and polynomial deductions in the ARDL model given in (1), follows

$$\Delta y_t = \mu - A(1) \left[y_{t-1} - \frac{B(1)}{A(1)} x_{t-1} \right] + \sum_{i=1}^{r-1} \delta_i \Delta y_{t-i} + \sum_{i=0}^{s-1} \gamma_i \Delta x_{t-i} + \varepsilon_t \quad (2)$$

where $A(L) = 1 - \sum_{i=1}^r \alpha_i L^i$ and

$$B(L) = \sum_{i=0}^s \beta_i L^i. \quad \text{The}$$

term $-A(1) \left[y_{t-1} - \frac{B(1)}{A(1)} x_{t-1} \right]$ is called error correction term, $B(1)/A(1)$ is the long-run multiplier, $-A(1)$ measures the adjustment speed short-run to equilibrium (long-run) and indicates the proportion of the equilibrium error which is reflected in the dependent variable for the following period.

According to [7], the parameterisation of (Eq. 1) to (Eq. 2) provides advantages in the estimation as immediate obtaining the coefficient $-A(1) = \phi$ adjustment short-run to equilibrium; also allowing the test of the presence of the error correction term ($H_0: \Phi=0$ versus $H_1: \Phi<0$). Another important advantage is that the author emphasizes that the estimation by the (Eq. 2) attenuates multicollinearity problems between the regressors, which is common in estimation with

(Eq. 1). Finally, as the ordinary least squares estimator is invariant linear transformations it is preferable to (Eq. 2).

2.2 The Bounds Testing Approach

The approach proposed by [8] part of the reformulation of the ADL model exposed in (Eq. 1). The procedure is adopted for several reasons. Firstly, the procedure is simpler than other techniques, such as multivariate cointegration Johansen and Juselius (1990). Secondly, the test is applicable regardless if it the regressors in the model are purely stationary, I(0), or purely integrated of order 1, I(1). However, the presence of some time series integrated order 2, the procedure fails. Third, the test is relatively more efficient in smaller sample sizes [9].

The first step in the method proposed by [8] is estimated by ordinary least squares (OLS), the unrestricted error correction model, known as ECM unrestricted given by

$$\Delta Y_t = \mu_1 + \theta_0 Y_{t-1} + \sum_{i=1}^m \theta_i X_{i,t-1} + \sum_{i=1}^{r-1} \delta_i \Delta Y_{t-i} + \left(\sum_{k=0}^{s_1-1} \gamma_{1k} \Delta X_{1,t-k} + \dots + \sum_{k=0}^{s_m-1} \gamma_{mk} \Delta X_{m,t-k} \right) + \eta_t \quad (3)$$

However, to select the order lags of model in (3), the information criterion is used, such as the AIC, in a model $ADL(r, s_1, \dots, s_m)$ associated with parameterized shape in (3), given by

$$Y_t = \sum_{i=1}^r \alpha_i Y_{t-i} + \sum_{k=0}^{s_1} \beta_{1k} X_{1,t-k} + \dots + \sum_{k=0}^{s_m} \beta_{mk} X_{m,t-k} + \mu + \zeta_t \quad (4)$$

After the estimation of the ECM unrestricted model (3), the second step is aimed at test the absence of long-run relationship between the variables. This is done by an F test for the joint significance of the coefficients of the series of lagged variables, that is, against the alternative that at least one is not null. Two bounds are supplied for testing the cointegration when the independent variables are I(0) or I(1): a lower value assuming that the regressors are I(0) and an upper limit assuming that the regressors are I(1). If the F statistic is greater than the upper critical value, the null hypothesis of absence of long-run relationship can be rejected, regardless of the integration orders for the time series, i.e., there is cointegration. On

the other hand, if the test statistic is less than the lower critical value, the null hypothesis can not be rejected and no cointegration. Finally, if the statistic falls between the lower and upper critical values, the result is inconclusive [9]. The two sets of critical values were reported in [8].

In the third and last step, we obtain the parameters of the dynamic model of short-run by estimating an error correction model associated with long-run estimates, this is called ECM restricted or ECM conditional. The lags in the ECM restricted model are the same as ECM unrestricted. The ECM restricted is given by

$$\Delta Y_t = \mu + \varphi v_{t-1} + \sum_{i=1}^{r-1} \beta_i \Delta Y_{t-i} + \left(\sum_{k=0}^{s_1-1} \delta_{1k} \Delta X_{1,t-k} + \dots + \sum_{k=0}^{s_m-1} \delta_{mk} \Delta X_{m,t-k} \right) + \varepsilon_t \quad (5)$$

where

$$v_{t-1} = Y_{t-1} - a_0 - a_1 X_{1,t-1} - a_2 X_{2,t-1} - \dots - a_m X_{m,t-1}$$

and the coefficients $a_i, i = 1, \dots, m$ are the estimates by ordinary least squares of the coefficients $\alpha_i, i = 1, \dots, m$ in the equation

$$Y_t = \alpha_0 + \alpha_1 X_{1,t} + \alpha_2 X_{2,t} + \dots + \alpha_m X_{m,t} + v_t \quad (6)$$

Note that (Eq. 5) differs from (Eq. 3) only in the error correction term. The restricted ECM restricts their coefficients to the long-run relationship in (6). In (Eq. 5) β, δ, γ_1 e α_1 are the dynamic coefficients of the short-run model that converges to equilibrium, where φ is the adjustment speed.

3. DATA AND METHODOLOGY

3.1 Data

The data used in the model are non-automated readings of COF20 sensor direct pendulum located in the concrete block type gravity relieved F19/20 Dam Hydroelectric Power Plant Itaipu, Brazil. The sensor is located in the quota 125,15m. Nearby surface thermometers to COF20 sensor are considered and whose data are available. The TSF14 installed in the quota 100.25 m, and TSF15 and TSF16 installed in the quota 150.85 m. The period chosen for analysis is from January 2000 to June 2015. During this period, there was no presence of missing data and frequency of readings was monthly.



The COF20 sensor measures displacement in the x and y direction. The direction x is the displacement of the direction of flow (upstream/downstream) and the y direction is perpendicular to flow (left bank/right bank). This application uses only displacements in the x direction, namely variable "cof20x".

Therefore, the regressors in the model are: the lags of the dependent variable cof20x, current and lagged values of surface thermometers (tsf14, tsf15 and tsf16) and current and lagged values of the reservoir level (level).

Initially, 186 observations were divided into two sets. The training set for model fitting, consisting of observation at the initial time (Jan/00) to the 180th observation (Dec/14) and set to forecast composite by observing 181st observation (Jan/15) to the 186th observation (Jul/15). For modeling was used Eviews 9 software.

3.2 Methodology

This study follows the basic steps of the methodology proposed by [8] with some additional test for prediction purposes. The steps are:

- I. check the degree of integration of each variable because if the variables are I(2) the method cannot be applied;
- II. formulate an ECM unrestricted model, determining the appropriate lag structure;
- III. validate the model with appropriate tests;
- IV. executing the Bounds Testing, in the ECM unrestricted, to check evidence of long-run relationship between the variables;
- V. if the result is positive in the previous step, estimating a long-run to the original model series;
- VI. formulate ECM restricted;
- VII. validate the model with appropriate tests;
- VIII. to forecast and build the confidence intervals, as well as evaluating the error.

The ECM restricted is given by

$$\begin{aligned} \Delta cof 20x_t = & \theta_1 cof 20x_{t-1} + \theta_2 tsf 14_{t-1} + \theta_3 tsf 15_{t-1} + \\ & + \theta_4 tsf 16_{t-1} + \theta_5 level_{t-1} + \sum_{i=1}^{r-1} \delta_i \Delta cof 20x_{t-i} + \\ & + \sum_{i=0}^{s_1-1} \gamma_i \Delta tsf 14_{t-i} + \sum_{i=0}^{s_2-1} \eta_i \Delta tsf 15_{t-i} + \\ & + \sum_{i=0}^{s_3-1} \lambda_i \Delta tsf 16_{t-i} + \sum_{i=0}^{s_4-1} \phi_i \Delta level_{t-i} + \\ & + \sum_{i=1}^{11} \sigma_i D_{it} + \mu + \varepsilon_t \end{aligned} \quad (7)$$

The monthly dummy variable (D_{it}) is included to capture seasonal effects, where D_{1it} representing January until D_{11it} representing November. If there is long-run relationship between the variables, the formulation from static relationship is given by

$$\begin{aligned} cof 20x_t = & \beta_0 + \beta_1 tsf 14_t + \beta_2 tsf 15_t + \beta_3 tsf 16_t + \\ & + \beta_4 level_t + \sum_{i=1}^{11} \sigma_i D_{it} + v_t \end{aligned} \quad (8)$$

And ECM restricted is given by

$$\begin{aligned} \Delta cof 20x_t = & \mu + \sum_{i=1}^{r-1} \delta_i \Delta cof 20x_{t-i} + \sum_{i=0}^{s_1-1} \gamma_i \Delta tsf 14_{t-i} + \\ & + \sum_{i=0}^{s_2-1} \eta_i \Delta tsf 15_{t-i} + \sum_{i=0}^{s_3-1} \lambda_i \Delta tsf 16_{t-i} + \\ & + \sum_{i=0}^{s_4-1} \theta_i \Delta level_{t-i} + \sum_{i=1}^{11} \sigma_i D_{it} + \lambda(v_{t-1}) + \varepsilon_t \end{aligned} \quad (9)$$

4. RESULTS AND DISCUSSION

Table 1 shows the test values ADF (Augmented Dickey-Fuller), known as unit root test, that is, under the null hypothesis series is not stationary. Considering the level of significance of 5%, it appears that the series cof20x, tsf16 and level are integrated of order one (I(1)) and the series tsf14 and tsf15 are stationary (I(0)). Thus, given the miscellany of levels of integration in the series has been the scene for the implementation of Bounds Testing approach [8], which shows robust results in the presence of series I(0) and I(1).

4.1 ECM Unrestricted

For the formulation of ECM unrestricted is necessary determine the optimal lag. Considering the Akaike Information Criterion (AIC), the



$ARDL(r,s_1,s_2,s_3,s_4) = ARDL(8,8,0,7,1)$ is the chosen model with a coefficient of determination (R^2) of 0.95.

Estimates of ECM unrestricted model and their p-values were omitted due to the paper page limit. The ECM unrestricted model does not impose restrictions on the constant and does not include the term trend, as the latter proved to be irrelevant. In the estimated some lags of variables may have been removed from statistical insignificance of the coefficients, however, no variable has been eliminated for purposes of model fit. In fact, with the removal of some of the statistically insignificant lags lost if any relevant property such as, for example, the absence of autocorrelation in the residuals.

About the diagnosis of residues ECM unrestricted model, the Jarque-Bera test suggests that the errors are normally distributed ($JB = 1.526$ (0.466)). The correlogram residues does not present significant correlations to the lag 36. The Breusch-Pagan-Godfrey and White test does not reject the null hypothesis that the errors have homoscedastic variance. As for the diagnosis of the model, the RESET test (Regression Specification Error Test) Ramsey with $F_{(1,131)}=1.748302$ and p-value of 0.0828 does not reject the null hypothesis of correct model specification. Under these terms the model meets the requirements for Bounds Testing of [8].

4.2 Bounds Testing

Estimate ECM unrestricted model goes to the verification phase the existence of cointegration, i.e., whether or not a long-run relationship between the temperatures and the reservoir level with the displacement of COF20 sensor in the direction of flow. For this calculate the F-test joint significant coefficients of the regressors delayed one period, namely $tsf14_{t-1}$, $tsf15_{t-1}$, $tsf16_{t-1}$, $cof20x_{t-1}$ and $level_{t-1}$. The null hypothesis of joint nullity of coefficients means there is no long-run equilibrium relationship. The value of the F-statistic was 5.7265 with a p-value of 0.0001. Using the critical values proposed by [8], the statistic value exceeds the upper limit even at the 1% level of significance which has as limits 3.74 and 5.06. So, the F-statistic indicates the existence of long-run equilibrium relationship by rejecting H_0 .

It is also important to test the nullity of $cof20x_{t-1}$ coefficient (ECM test) against the alternative that it

is less than zero corresponding to a necessary condition for stability of the model. The value of the t-statistic is -3.3835 with p-value of 0.0009, i.e., it rejects H_0 reinforcing the result of the test F which supports the hypothesis of long-run equilibrium relationship.

4.3 ECM Restricted

For ECM restricted (Eq. 9) is necessary to estimate the static relationship (Eq. 8), that is, estimate the long-run equilibrium relationship coefficients. In this model, the parameters are consistently estimated by ordinary least squares, however, since the variables involved can be $I(1)$, the usual inference procedures are not necessarily valid.

The estimated restricted error correction model (ECM restricted) with residues obtained from the model of long-run equilibrium relationship (Eq. 8). Table 2 shows the results of ECM restricted. The regressors in Table 2 have significant capacity to explain $\Delta COF20X$, since the F test rejects the global null hypothesis significance joint statistical insignificance of the estimates (F-statistic 20.5539 with p-value of 0.0000). Some coefficients present statistical insignificance at 5%, it was maintained over again to improve the model specification.

The coefficient of determination (although not of particular relevance for the regression analysis) indicate that just over 84% of the variation of the first difference of the displacements obtained by COF20 sensor in the flow direction (X) is explained by the regressors considered in the specification model, that is, is explained by the current and past values of the closest surface thermometers, the reservoir level and its own past values. The estimated coefficient adjustment short-run (-0.08025) is statistically significant and shows the sign (negative) expected, which shows the trend to reach the the long-run equilibrium relationship.

Table 3 follows the diagnostic tests ECM restricted model. The RESET test does not reject the null hypothesis of correct model specification and Breusch-Godfrey test can not detect serial correlation up to order 20. ARCH and Breusch-Pagan tests also do not reject the null hypothesis homoscedastic variance. In addition, the Jarque-Bera normality test is favorable the null hypothesis of normality residues.



In time series models is important to analysis the stability of the model structure. This analysis is performed by means of parameters of stability testing. This test is particularly important in this application because the forecast based on historical data may be invalid if any changes alter the relationship between the variables. In other words, if the relationship between variables is sensitive to any changes, then the model can not be used to forecast [10].

For the stability analysis was done using the QLR test (Quandt Likelihood Ratio). The QLR test did not reject the null hypothesis of no breaks in the range $[(0.15 \ 0.85)]$ of the sample. Follows in Figure 1 the graph of the values predicted by the model, observed values and residuals.

4.4 Forecast

To carry out the forecast, first partitions the sample leaving the last six observations of the original sample (Jan/2015 to Jul/2015) out. Then the models were estimated with the sample Jan/2000 to Dec/2014. With ECM restricted model estimated forecasts were simulated for truncated period. The forecast should be carefully evaluated because the presence of variables with a lag of the dependent variable. There are some possibilities, however, is limited here to describe the two types of prediction that will be compared, dynamic and static prediction.

The dynamics forecast is to predicting multi-step ahead from the start of prediction sample, i.e., the forecasts for subsequent observations will use amounts previously predicted the dependent variable. While the static prediction is realized forecasts one-step ahead of the dependent variable, so for each observation in the sample forecast calculation always uses the current value of the lagged dependent variable. Both methods produce similar results in the first period to a range forecasting. And finally, the prediction errors were calculated.

The statistics used to calculate the forecast error was the Root Mean Squared Error (RMSE), as the Mean Square Error (MSE) is sensitive to the presence of extreme predictions and Mean Absolute Percent Error (MAPE). Follow the formulas of RMSE and MAPE, where h is the forecast horizon, i.e., the number of future values that you want to predict.

Table 4 shows the values of the displacements observed in the COF20 sensor in the direction of flow, the values predicted by the ECM restricted in the dynamic and static approaches to $h = 6$, the confidence intervals and measures evaluation of errors.

Note that until the forecast for Mar/2015, the dynamic approach performed better than static, and, from Apr/2015 forecast, the static approach excelled. This is consistent because in the dynamic approach to forecast several steps ahead is by definition a sequence of predictions one step ahead considering them as observed values (when in fact they are not), so the higher the forecast horizon, the greater the error in the final amount, since the final error is an "accumulation" of the various errors of the predictions made a step forward.

It is apparent that, from a certain number of steps forward error of the predicted value becomes so large that the prediction comes to have little meaning. There is no definitive solution to this, so the forecast horizon should therefore be less than or equal to a value whose final prediction error is acceptable. It is recommended that error and performance measures are discussed in each application. Figure 2 shows the predicted values, the observed values and the lower and upper limits using the standard error of the mean (± 2 SE).

It is recommended also that the dynamic model is reviewed periodically in order to incorporate new effects of climatic variation.

5. CONCLUSION

The objective of this paper is to present a model for forecasting of horizontal displacement of a block concrete on a dam under temperature variation effects and reservoir level variation. The forecast of these displacements and the confidence intervals for these estimates provides a contribution to the technicians and engineers in decision-making with regard to the monitoring of this block.

The ADL model and Bounds Testing of [8] formed the structure of this empirical study. The result of the Bounds Testing, robust to the presence of series $I(0)$ or $I(1)$ clearly favors the hypothesis of long-run equilibrium relationship as it is statistically significant even at 1%.

The coefficient of determination indicates that just over 84% of the variation of the first difference



of displacement is explained by current and past values of the first differences of the closest surface thermometers, the reservoir level and its own past values. The estimate of the short-run adjustment coefficients (-0.08025) is statistically significant and shows the sign expected (negative), which shows the trend to reach the long-run equilibrium relationship.

The final model (ECM restricted) passed through various diagnostic tests like the RESET test, Breusch- Godfrey tests, ARCH Breusch- Pagan, Jarque-Bera and QLR thus the model could be used for prediction purposes.

The forecast was performed for the period January 2015 to June 2015, the dynamic and static approach. To the horizon of this forecast ($h = 6$ months), a static approach is preferable, however, the dynamic approach is more interesting, since whether to perform predictions which are not limited to one step forward. For the prediction help in monitoring block will be considered a forecast horizon equal to $h = 3$ months, where the two approaches do not significantly differ among themselves and neither the actual observed value, that is, the dynamic model is revised every 3 months by updating the confidence limits and also incorporating new effects of climatic variation.

In future work we intend to analyze the displacement in the y direction which is perpendicular to the flow (right bank / left bank) for the same sensor and also extend the modeling to all sensors block.

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Table 1: ADF unit root test

Variables	t-statistics
cof20x	-1.42
Δ cof20x	-3.75*
tsf14	-4.00*
tsf15	-4.64**
tsf16	-2.64
Δ tsf16	-9.34**
level	-1.27
Δ level	-7.96**

Note: “**” and “*” denote reject H_0 the level of significance of 1% and 5%, respectively.

Table 2: Restrict ECM Model

Variable	Coefficient	p-value	Variable.	Coefficient	p-value
μ	-0.71	0.00	Δ tsf16 _{t-1}	-0.04	0.00
ν_{t-1}	-0.08	0.03	Δ tsf16 _{t-2}	-0.03	0.03
Δ cof20x _{t-1}	-0.31	0.00	Δ tsf16 _{t-3}	-0.02	0.25
Δ cof20x _{t-2}	-0.16	0.06	Δ tsf16 _{t-4}	-0.02	0.20
Δ cof20x _{t-3}	-0.30	0.00	Δ tsf16 _{t-5}	-0.03	0.02
Δ cof20x _{t-4}	-0.25	0.01	Δ tsf16 _{t-6}	-0.02	0.17
Δ cof20x _{t-5}	-0.12	0.16	Δ level _t	0.15	0.00
Δ cof20x _{t-6}	0.06	0.49	jan	-0.20	0.27
Δ cof20x _{t-7}	-0.09	0.26	feb	0.14	0.51
Δ tsf14 _t	0.03	0.36	mar	0.48	0.06
Δ tsf14 _{t-1}	0.02	0.54	apr	1.06	0.00
Δ tsf14 _{t-2}	-0.05	0.09	may	1.09	0.00
Δ tsf14 _{t-3}	-0.05	0.10	jun	1.20	0.00
Δ tsf14 _{t-4}	-0.01	0.69	jul	1.62	0.00
Δ tsf14 _{t-5}	-0.02	0.52	aug	1.44	0.00
Δ tsf14 _{t-6}	-0.01	0.78	sep	1.05	0.00
Δ tsf14 _{t-7}	0.01	0.76	oct	0.55	0.03
Δ tsf16 _t	-0.04	0.00	nov	0.32	0.08

Table 3: Diagnostic tests of ECM restricted model

Tests	Statistics	p-value
Breusch-Godfrey	27.64	0.12
ARCH	0.19	0.91
Breusch-Pagan	43.88	0.14
Jarque-Bera	1.314	0.52
RESET	0.45	0.64

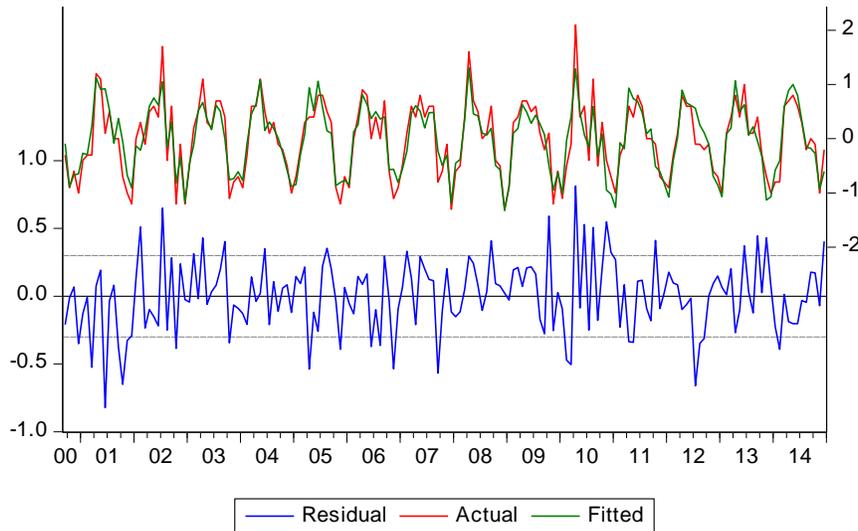


Figure 1: Graph of observed values, predicted and residues of ECM restricted model (dependent variable $\Delta cof20x$)

Table 4: Observed values, predicted values, confidence interval (C. I.) and errors of assessment measures.

Month	Observed	Dynamic	C. I.	t statistic	C. I.
01/15	6.8	6.90	[6.2,7.6]	6.90	[6.2,7.61]
02/15	6.5	6.51	[5.6,7.4]	6.41	[5.81,7.02]
03/15	6.9	7.06	[6.1,8.0]	7.08	[6.48,7.69]
04/15	7.4	7.60	[6.6,8.6]	7.43	[6.85,8.02]
05/15	7.8	8.44	[7.4,9.5]	8.31	[7.65,8.96]
06/15	8	8.74	[7.6,9.9]	8.14	[7.47,8.8]
	RMSE	0.42		0.23	
	MAPE	4.01		2.35	

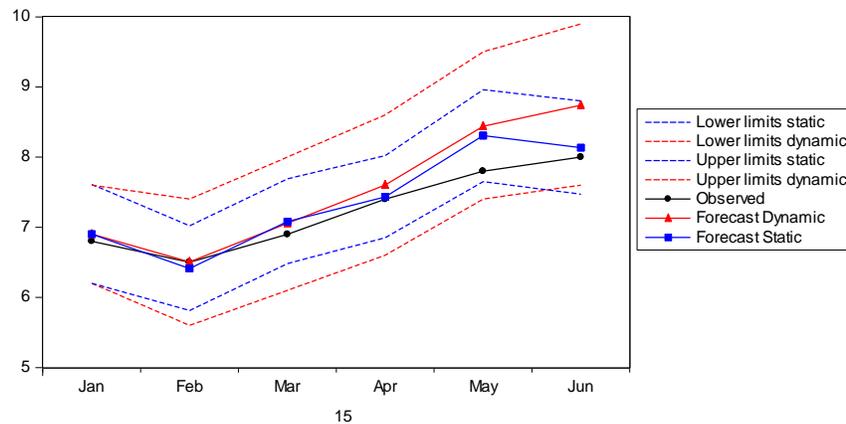


Figure 2: Graph of observed values, predicted values and confidence limits out of the sample.