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MODELING FLUID FLOW IN OPEN CHANNEL WITH HORSESHOE CROSS – SECTION

[1]J. JOMBA, [2]D.M.THEURI, [2]E. MWENDA, [2]C. CHOMBA

ABSTRACT

Flow in a closed conduit is regarded as open channel flow, if it has a free surface. This study considers unsteady non-uniform open channel flow in a closed conduit with horseshoe cross-section. We investigate the effects of the flow depth, channel radius, slope of the channel, manning constant and lateral inflow on the flow velocity as well as the depth at which flow velocity is maximum. The finite difference approximation method is used to solve the governing equations because of its accuracy, stability and convergence followed by a graphical presentation of the results. It is found that for a given flow area, the velocity of flow increases with increasing depth. Moreover, increase in the slope of the channel leads to an increase in flow velocity whereas increase in manning constant, radius of the channel and lateral inflow leads to a decrease in flow velocity.

This study goes a long way in controlling floods, irrigation and in construction of channels.

Key words: Open channel, velocity, depth.

1. INTRODUCTION

Water flows more rapidly on a steeper slope than on a gentle slope, but for a constant slope, the velocity reaches a steady value when the gravitational force is equal to the resistance of the flow. Over years, man has endeavored to direct water to the desired areas such as farms, where it is used for irrigation. He has also tried to draw water from storage sites such as reservoirs, dams and lakes. To achieve this objective, he has constructed open channels which are physical systems in which water flows with a free surface.

The cross-section of these channels may be open or closed at the top. The structures with closed tops are referred to as closed conduits while those with open tops are called open channels. This study focuses on a water flow in a open conduit with horseshoe cross-section. The findings of this study will go a long way in providing reference for designers of open channels and guidelines for the hydraulic analysis. It will also provide an understanding into the propagation of the rise and fall of water levels in natural rivers, originating from torrential rains or of breaking of control structures.

The earliest study on open channels was carried out by a French engineer called Chezy in 1768. He discovered the Chezy's formula and Chezy's constant. He was concerned with canal flows in his country France (Hamil,1995). Chezy's formula did not provide results that satisfied engineers.

Manning discovered the Manning formula (Chow, 1973). He identified the coefficient of roughness called the Manning Coefficient, which takes into account the bed materials, degree of channel irregularity, variation in shape and size of the channel and relative effect of channel obstruction, vegetation growing in the channel and meandering, Chadwick (1993).

In 1973, Chow carried out a study on open channel flows and developed many relationships such as velocity formula for open channel flows (Chow,1973)

Sinha and Aggarwal (1980) investigated the development of the laminar flow of a viscous incompressible fluid from the entry to the fully developed situation in a straight circular pipe.

They observed that velocity increases more rapidly during the initial development of the flow in comparison to the downstream flow. It was observed that during the initial stages of the development of the flow, the rate of increase in stream wise velocity is larger and consequently the pressure drop is larger in comparison with their values further downstream. Rantz (1982) studied open channel flows and developed a method of measuring discharge in shallow and deep channels.

Nalluri and Adepoju (1985) analyzed experimental data on resistance to flow in smooth channels of circular cross- section. The results of the tests

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showed that the measured friction factors are larger than those of a pipe with equivalent diameter.

Crossley (1999) investigated strategies developed for the Euler's equations for application to the Saint Venant equations of open channel flow in order to reduce run times and improve the quality of solutions in the regions of discontinuities. Carlos and Santos (2000) considered an adjoint formulation for the non-linear potential flow equation.

Makhanu (2001) worked on development of simple hydraulic performance model of Sasumua Pipelines of Nairobi. Tuitoek and Hicks(2001) modeled unsteady flow in compound channels with an aim of controlling floods.

Kwanza et al (2007) carried out investigations on the effects of the channel width, slope of the channel and lateral discharge for both rectangular and trapezoidal channels. They noted that discharge increases as the specified parameters are varied upwards and that trapezoidal channels are more hydraulically efficient than rectangular ones.

Sturm,T. and King,D. (1988) carried out a study on shape effects on flow resistance in horseshoe conduits where he found that friction factor increase approximately 20 percent in comparison to values taken from moody diagram for depths larger than half –full.

Mingliang Zhang et al(2012) studied depth-averaged modeling of free surface flows in open channels with emerged and submerged vegetation where model formulation the vegetation resistance was treated as momentum sink and represented by a manning type equation.

Ben Chie Yen, F.Asce (2002) studied open channel flow resistance where he discussed the differences between momentum and energy resistances between point, cross-sectional and resistance coefficients, as well as compound / composite channel resistance.He also discussed the issue of linear separation approach versus non linear approach to alluvial channel resistance.

Hancua et al (2001) studied the numerical modeling and experimental investigations of the fluid flow and contaminant dispersion in channels where he presented results for both a computational and a physical model of the fluid and mass transport processes in a straight channel with flow control obstructions being placed in the channel.

Francisco J.M. Simoes (2010) studied flow resistance in open channels with fixed and movable bed where by a new resistance law for surface (grain) resistance, the resistance due to the flow viscous effects on the channel boundary roughness elements was presented for cases of flow in the transition ($5< Re^{*}<70$) and fully rough ($Re^{*}\geq 70$) turbulent flow regimes, where Re^{*} is the Reynolds number based on shear velocity and sediment particle mean diameter.

He also showed that the new law was sensitive to bed movement without requiring previous knowledge of sediment transport conditions.

Mohammad R. Hashemi et al(2007) studied a differential quadrature analysis of unsteady open channel flow whereby saint-venant equations and the related non homogenous, time dependent boundary conditions are discretized in spartial and temporal domain.

Jiliang et al (2010) studied iterative formulas for normal depth of Horseshoe cross sections where he presented equations of geometric elements and derives iterative formulas for calculating the normal depth for all types of standard horseshoe cross sections. He based on the principle of gradual optimization fitting, general estimation formulas were also developed for direct computation of the normal depth for all types of the standard horseshoe cross sections. The formulas had a high accuracy with a relative error of less than 0.5%.

JunkeGuo and Pierre Y.Jullen (2005) studied shear stress in smooth rectangular open channel flows. He discovered that the shear stresses are function of three components gravitational, secondary flows and interfacial shear stress.

Cheng L.C et al (1996) studied mathematical hydraulics of surface irrigation whereby he discovered that the kinematic-wave method is the most simplified method so far as it can be deduced from the general equations without losing too much the generality of the formation of the mathematical model for the flow of water-waves in the surface irrigation.

M.N. Kinyanjui et al (2012) studied modeling fluid flow in an open channel with circular cross-section. They established that for a fixed flow area, the flow velocity increases with increase in depth from the bottom of the channel to the free stream and that maximum velocity occurs just below the free surface .Also reduction in the slope leads to a decrease in the flow velocity.

The current study is on investigation of effects of parameters such as channel radius, channel slope, manning constant and lateral inflow on velocity. Investigation of variation of velocity with depth is also carried out.

This would solve the problem of flooding during heavy rains as well as the shortage of water for irrigation

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1.1 MATHEMATICAL ANALYSIS

Governing equations

The Continuity and the Momentum equations may be used to analyze open channel flow. In this study the flow is unsteady, non uniform, incompressible and laminar. From M.N Kinyanjui et al (2012), the continuity equation is given by $\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial x} - u = 0$ (1)

which is expanded to $qT\frac{\partial y}{\partial x}+A\frac{\partial q}{\partial x}+T\frac{\partial y}{\partial t}-u=0$ (2)

Horseshoe cross section is divided into three zones of flow depth. Merkley (2005), calculated y_1, y_2 and y_3 as shown below:

$$y_1 = r\left[1-\left(\frac{1+\sqrt{7}}{4}\right)\right]$$
$$y_2 = \frac{r}{2} - y_1 = \frac{r}{2} - r\left[1-\left(\frac{1+\sqrt{7}}{4}\right)\right] \qquad (3)$$
$$y_3 = \frac{r}{2}$$

Equations related to cross sectional area, perimeter and water surface width are not the same (Merkley, 2005). For $0\leq y\leq y_1$, These equations are as follows

Cross section

$$A = (y-r)\sqrt{y(2r-y)} + r^2\left[\sin^{-1}\left(\frac{y-r}{r}\right)+\frac{\pi}{2}\right]$$

Topwidth (4)

$$T_w = 2r\sqrt{1-(1-\frac{y}{r})^2}$$

Substuting equations (3) and (4) in the continuity equation (2) we have

$$q2r\sqrt{1-(1-\frac{y}{r})^2}\frac{\partial y}{\partial x}+\left\{(y-r)\sqrt{y(2r-y)}+r^2\left[\sin^{-1}\left(\frac{y-r}{r}\right)+\frac{\pi}{2}\right]\right\}\frac{\partial q}{\partial x}+2r\sqrt{1-(1-\frac{y}{r})^2}\frac{\partial y}{\partial t}-u=0 \qquad (5)$$

This is the continuity equation for horseshoe cross-section whose the depth y of the flow is $0\leq y\leq y_1$.

The momentum equation is given by Yen 1973

$$\frac{g}{\beta}(S_o-S_f)-\frac{qu}{A}=\frac{\partial q}{\partial t}+q\frac{\partial q}{\partial x}+\frac{g}{\beta}\frac{\partial y}{\partial x} \qquad (6)$$

The cross-sectional area of this study is a uniform horseshoe cross-section, thus the value of momentum coefficient β is equal to one. Replacing it in equation (6) we have

$$g\left(S_o-\frac{n^2q^2}{R^{\frac{4}{3}}}\right)-\frac{qu}{A} = \frac{\partial q}{\partial t}+q\frac{\partial q}{\partial x}+g\frac{\partial y}{\partial x} \qquad (7)$$

Solution procedure

The governing equations (5) and (7) are solved using diffusing difference approximation as proposed by viessman et al. These equations are given as follows

$$y^*_{i,j+1}=0.5(y^*_{i-1,j}+y^*_{i+1,j}) -\Delta t\left\{\frac{(y-r)\sqrt{y(2r-y)}+r^2\left[\sin^{-1}\left(\frac{y-r}{r}\right)+\frac{\pi}{2}\right]}{2rL\sqrt{1-(1-\frac{y}{r})^2}}\frac{q^*_{i+1,j}-q^*_{i-1,j}}{2\Delta x}+q^*_{i,j}\frac{y^*_{i+1,j}-y^*_{i-1,j}}{2\Delta x}-\frac{u}{2rv\sqrt{1-(1-\frac{y}{r})^2}}\right\} \qquad (8)$$

$$q^*_{I,j+1}=0.5(q^*_{i-1,j}+q^*_{i+1,j}) - \Delta t\left\{q^*_{i,j}\frac{q^*_{i+1,j}-q^*_{i-1,j}}{2\Delta x}+\frac{g}{Fr}\frac{y^*_{i+1,j}-y^*_{i-1,j}}{2\Delta x}+\frac{u}{Fr(y-r)\sqrt{y(2r-y)}+r^2\left[\sin^{-1}\left(\frac{y-r}{r}\right)+\frac{\pi}{2}\right]}q^*_{i,j}-\frac{g}{Fr}\left[S_0-\frac{n^2v^2}{2R^{\frac{4}{3}}}\left(q^{*2}_{i-1,j}q^{*2}_{i+1,j}\right)\right]\right\} \qquad (9)$$

subject to the initial conditions $y^*(x^*,0) = 0.5$, $q^*(x^*,0) = 10$, for all $x^* > 0$ and boundary conditions $y^*(0,t^*) = 0.5$ $q^*(0,t^*) = 10$, for all $t^* > 0$.

1.II RESULTS AND DISCUSSION

The values for velocity, q against those of the depth y were plotted giving the curve in figure 1 below. Furthermore, by varying the specified parameters, the curves figures 2 to 7 were obtained.

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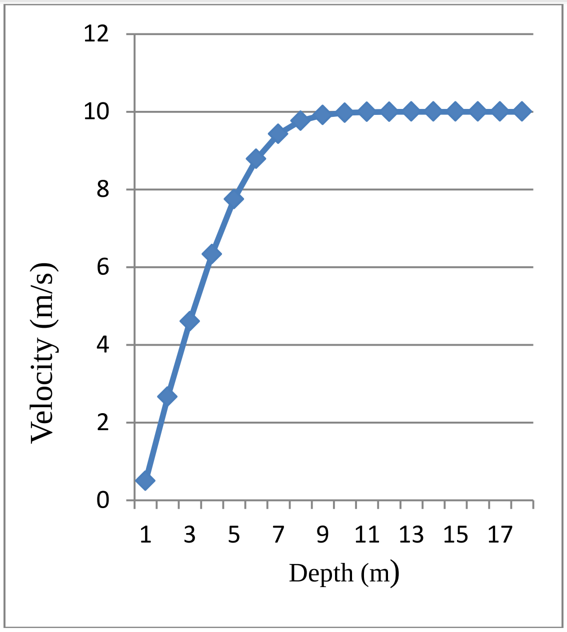

FIGURE 1:VELOCITY PROFILE VERSUS DEPTH
r=4, P=4.0429, s=0.004, n=0.012,u=0.002

From figure 1, we observe that the velocity increases with increase in depth. The free surface occurs at a depth of 10 m and the velocity of the fluid layer at this depth is 10 m/s. The flow velocity in a channel varies from one point to another due to shear stress at the bottom and at the sides of the channel.

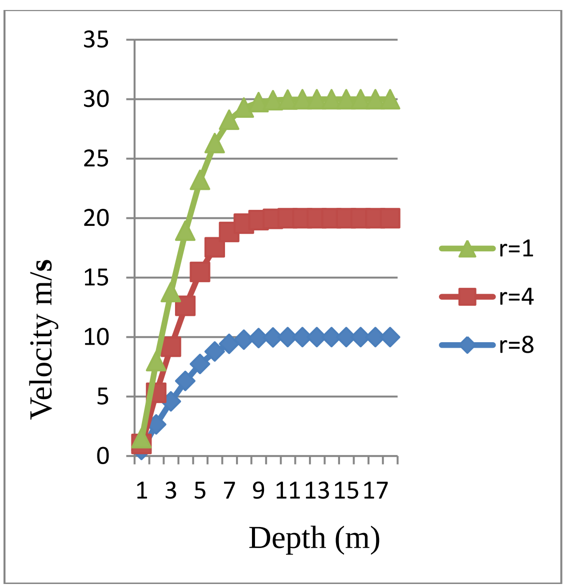

FIGURE 2: EFFECT OF CHANGE OF CHANNEL RADIUS ON VELOCITY PROFILE. s=0.004, n=0.012, u=0.002

Figure 2, we observe that increasing the radius from 1m to 4m results to a decrease in the flow velocity. An increase in the radius results in an increase in the wetted perimeter because the fluid will spread more in the conduit. A large wetted perimeter will result to a large shear stresses at the sides of the channel and therefore the flow velocity will be reduced. Moreover, an increase in the cross-sectional area of flow leads to an increase in wetted perimeter hence reducing velocity of the channel. This is because the radius of the channel is directly proportional to cross-sectional area of the channel.

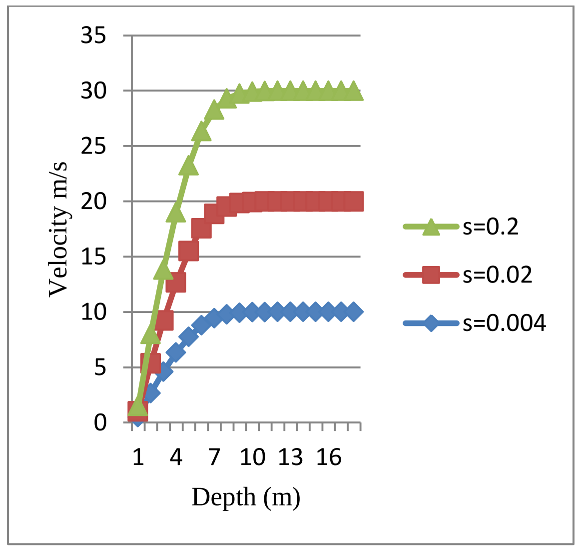

Figure 3: EFFECT OF CHANGE OF SLOPE ON VELOCITY PROFILE. n=0.012, u=0.002

From figure 3, we observe that a reduction in the slope from 0.2 to 0.004 leads to a decrease in the flow velocity. The velocity formula equation shows a direct relationship between flow velocity and the slope. Thus a decrease in slope results to a decrease in the flow velocity. Decrease in the slope causes the center of gravity to move down causing stability in the water molecules.

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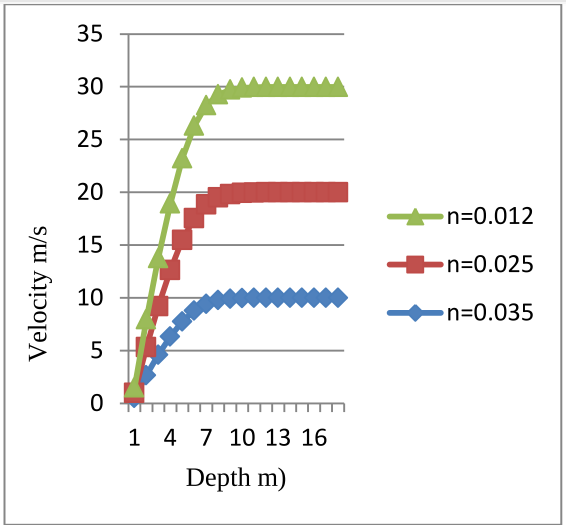

Figure 4: EFFECT OF CHANGE OF MANNING CONSTANT ON velocity profiles. r=4,A=1.3080,P=4.0429 ,n=0.012, u=0.002

Figure 4, we observe that increasing the manning constant results to a decrease in the flow velocity. An increase in the manning constant results to large shear stresses at the sides of the channel.

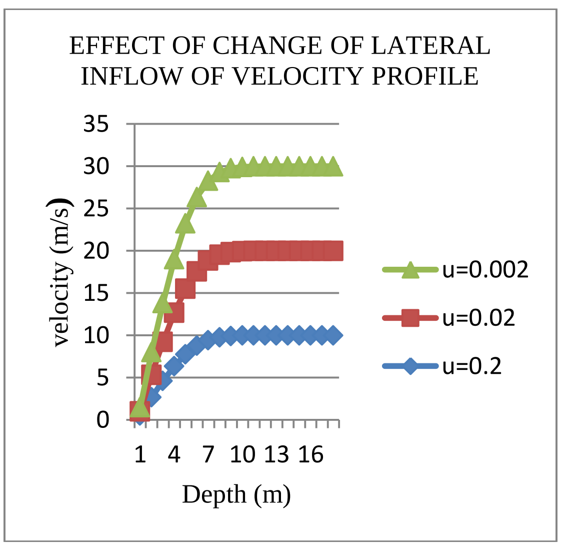

Figure 5: EFFECTS OF CHANGE OF LATERAL INFLOW OF VELOCITY PROFILE. r=4,A=1.3080,P=4.0429,s=0.004,n=0.012,

Figure 5, shows that a decrease in the lateral inflow rate per unit length of the channel from 0.2 to 0.002 leads to increase in the channel velocity. An increase in the lateral inflow rate per unit length of the channel means an increase in the amount of flow per unit time. This leads to increase in the cross-sectional area of which leads to increase in the wetted perimeter that increases the shear stress between the sides and bottom of the channel with the fluid particles leading to decrease in the flow velocity.

I.III CONCLUSION

The objective of this thesis was to investigate the effects of the various flow parameters on the flow velocity. An analysis of the effects of the various parameters on the flow velocity has been carried out. The equations governing the flow considered in the problem are non-linear and therefore to obtain their solutions, an efficient finite difference scheme has been developed. The mesh used in the problem considered in this work is divided uniformly.

The various flow parameters were varied one at a time while holding the other parameters constant. This was repeated for all the flow parameters and the results presented graphically. It was established that for a fixed flow area the flow velocity increases with increase in depth from the bottom of the channel to the free stream. The flow velocity in a channel varies from one point to another due to shear stress at the bottom and at the sides of the channel.

An increase in the radius of the conduit results to a reduction of the flow velocity. This is because, as the radius is increased, so is the wetted perimeter as the fluid spreads more in the conduit. A large wetted perimeter will result to large shear stresses at the sides of the channel and therefore the flow velocity will be reduced. Moreover, an increase in the roughness coefficient also results to a decrease in the flow velocity due to large shear stresses at the sides of the channel. This means that the motion of fluid particles at or near the surface of the conduit will be reduced. The velocity of the neighbouring molecules will also be reduced due to constant bombardment with the slow moving molecules leading to an overall reduction in the flow velocity.

Reduction in the slope leads to a decrease in the flow velocity since the slope and flow velocity are directly proportional. This is true as reflected by both the Chezy and Manning formulae discussed in chapter two. An increase in the cross sectional area of flow results to a decrease in the flow velocity. An increase in the cross-sectional area of flow leads to an increase in the wetted perimeter. A large wetted perimeter results to high shear stresses at the sides of the channel which results to a reduction in the flow velocity.

An increase in the roughness coefficient results to large shear stress at the sides of the channel. This means that the motion of the fluid particles at or

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near the surface of the conduit will be reduced. The velocity of the neighboring molecules will also be lowered due to constant bombardment with the flow moving molecules leading to an overall reduction to the flow velocity. A decrease in the lateral inflow rate per unit length of the channel leads to an increase in the channel velocity. An increase in the lateral inflow rate per unit length of the channel means an increase in the amount of flow per unit time. This leads to increase in the cross-sectional area of which leads to increase in the wetted perimeter that increases the shear stress between the sides and bottom of the channel with the fluid particles leading to decrease in the flow velocity.

1.IV RECOMMENDATIONS

The results obtained in this study are theoretical. Therefore it is recommended that an experimental approach to this problem be undertaken taken and compare the results with the theoretical ones. This calls for more research. It is also recommended that further research should be carried out on

- Effects of lateral outflow on discharge
- Turbulent flow of the same cross-sectional shape
- Fluid flows through elliptic channels
- Use of implicit finite difference technique

1.V NOMENCLATURE

q	Mean velocity of flow (m/s)
L	Length of the channel (m)
Q	Discharge (m^3s^{-1})
g	Acceleration due to gravity (ms^{-2})
A	Cross-section area of flow (m^2)
n	Manning coefficient of roughness ($m^{-1/3}s$)
s_o	Slope of the channel bottom
s_f	Friction slope
P	Wetted perimeter (m)
T_w	Top width of the free surface (m)
y	Depth of the flow (m)
t	Time (s)
u	Uniform inflow (m^2s^{-1})
Fr	Froude number (dimensionless)
Re	Reynolds number (dimensionless)
ρ	Density (kgm^{-3})

1.VI REFERENCES

1. Ben, C et al, *open channel flow resistance*. Journal of hydraulic engineering, vol. 128, No. 1, 2002, pp 20- 29.
2. Chadwick, A. and Morfeit, J. *Hydraulics in Civil Engineering and environmental Engineering*, Chapman & Hall, 1993, pp. 187-200.
3. Cheng. L et al. *mathematical hydraulics of surface irrigation*. Journal of hydraulics division. V. 106, 1996, pp 1021-1040.
4. Chow V.T., *Open Channel Flows*, Mc Graw- Hill, 1973, pp. 1-60.
5. Crossley, A. .J. *Accurate and efficient numerical solutions for Saint Venant Equations of open channels*, Ph .D Thesis, University of Nottingham, UK. 1999.
6. Franciso, j et al. *Flow resistance in open channels with fixed and movable bed.* USGS geomorphology and sediment transport laboratory.2010.
7. Gupta,V. and Gupta *S.K. Fluid Mechanics and its Applications*, Wiley Eastern, 1984, Pp.87-144.
8. Hamil, L. *Understanding hydraulics*, Mc Millan, 1995, Pp. 200 - 273.
9. Hancua et al , *Numerical modeling and experimental investigations of the fluid flows.* Journal for science and engineering, v. 35
10. Jiliang,F et al. *Iterative formulas for normal depth of horseshoe cross-sections.* Flow measurement and instrumentation, 2010, pp 43-49.
11. Junke, G et al. *shear stress in smooth and rectangular open channel.* Civil engineering faculty publication. Paper 1.2005.
12. Kinyanjui, J. K et al,*modeling fluid flow in open channel with circular cross-section.* Journal of agricultural science and technology. V 12, 2012, pp 80-91.