Jan. 2015. Vol. 6. No. 01 ISSN2305-8269
International Journal of Engineering and Applied Sciences

www.eaas-journal.org

STRING COSMOLOGICAL MODELS IN BIANCHI TYPE-III SPACE-TIME WITH BULK VISCOSITY AND Λ TERM

[1]PREETI SONI, [2]SAPNA SHRIMALI

[1]Rsearch Scholar, Department of Mathematics, Pacific Academy of Higher Education & Research University, Udaipur (Rajasthan),India
[2]Assoc. Prof., Department of Mathematics, Pacific Academy of Higher Education & Research University, Udaipur (Rajasthan),India
Email: [1]preeti100ni@gmail.com, [2]shrimalisapna@gmail.com

ABSTRACT

The present study deals with exact solution of Einstein' field equations in Bianchi type-III string cosmological models with bulk viscosity and variable cosmological term Λ. Exact solutions of the field equations are obtained by assuming the conditions: the coefficient of the viscosity is proportional to the expansion scalar, $\xi \propto \theta$, expansion scalar is proportional to shear scalar, $\theta \propto \sigma$, and Λ is proportional to the Hubble parameter. The physical implications of the models are also discussed in detail.

Keywords*: Bianchi-III space-time, Bulk Viscosity, Variable Λ, expansion scalar, shear, Hubble parameter*

1. INRODUCTION

Cosmic string in recent years have drawn considerable attention among researchers for various aspects such that the study of the early universe. It is well known that in an earlier stage of the universe when the radiation in the form of photons as well as neutrinos decoupled from matter, it behaved like a viscous fluid. Misner [1, 2] have studied the effect of viscosity on the evolution of cosmological models. Nightingale [3] has investigated the role of viscosity in cosmology. Thus, we should consider the presence of a material distribution other than a perfect fluid to have realistic cosmological models. Heller and Klimek [4] emphasized that the introduction of bulk viscosity effectively removes the initial singularities with in a certain class of cosmological models. The effect of bulk viscosity on the cosmological evolution has been studied by a number of authors in the fram work of general theory of relativity viz. Pavon et al. [5], Maartens [6], Kalyani and Singh [7], Singh et al. [8].

On other hand, one outstanding problem in cosmology is the cosmological constant problem [9, 10]. Since, its introduction and its significance has been studied from time to time by various workers [11-13]. In modern cosmological theories, the cosmological constant remains a focal point of interest. A wide range of observations now suggests compellingly that the universe possesses a non-zero cosmological constant [14]. Recently, Singh et al. [15] investigated Bianchi type-III cosmological models with gravitational constant G and the cosmological constant Λ.

In this chapter, we have investigated Bianchi type-III string cosmological models with bulk viscosity and cosmological term Λ(t). To obtain an explicit solution, we assume that the coefficient of the viscosity is proportional to the expansion scalar $\xi \propto \theta$, and the expansion scalar is proportional to the shear scalar, $\theta \propto \sigma$, and for the cosmological term Λ, we assume that it is proportional the Hubble parameter $\Lambda \propto H$.

International Journal of Engineering and Applied Sciences

www.eaas-journal.org

2. DERIVAITON OF FIELD EQUATION

A. THE GRAVITATIONAL FIELD:

In our case, we consider the Bianchi type-III metric in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{-2\alpha x}dy^2 + C^2dz^2 \quad (1)$$

where A, B and C are functions of time alone and a is a constant. The metric (1) has the following non-zero christoffel symbols.

$$\Gamma^1_{14} = \frac{\dot{A}}{A},$$

$$\Gamma^1_{22} = \frac{\alpha B^2}{A^2}\mathrm{e}^{-2\alpha x},$$

$$\Gamma^2_{12} = -\alpha$$

$$\Gamma^2_{24} = \frac{\dot{B}}{B}$$

$$\Gamma^3_{34} = \frac{\dot{C}}{C},$$

$$\Gamma^4_{11} = A\dot{A}$$

$$\Gamma^4_{22} = B\dot{B}e^{-2\alpha x},$$

$$\Gamma^4_{33} = C\dot{C}.$$

Where dots on A, B, represents the ordinary differentiation with respect to *t*.
The surviving components of the mixed Ricci tensor R_i^j are as follows:

$$R_1^1 = \frac{\alpha^2}{A^2} - \frac{\ddot{A}}{A} - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{A}\dot{C}}{AC}, \quad (2a)$$

$$R_2^2 = \frac{\alpha^2}{A^2} - \frac{\ddot{B}}{B} - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}\dot{C}}{BC}, \quad (2b)$$

$$R_3^3 = -\frac{\ddot{C}}{C} - \frac{\dot{A}\dot{C}}{AC} - \frac{\dot{B}\dot{C}}{BC}, \quad (2c)$$

$$R_4^4 = -\frac{\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\ddot{C}}{C}, \quad (2d)$$

$$R_1^4 = \alpha\left(\frac{\dot{B}}{B} - \frac{\dot{A}}{A}\right), \quad (2e)$$

From (2) one finds the following Ricci scalar for the Bianchi type-IIII universe,

$$R = -2\left[\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{\alpha^2}{A^2}\right] \quad (3)$$

The non-vanishing components of Einstein tensor $G_i^j = R_i^j - \frac{1}{2}R$, are given by

$$G_1^1 = \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC}, \quad (4a)$$

$$G_2^2 = \frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC}, \quad (4b)$$

$$G_3^3 = \frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\alpha^2}{A^2}, \quad (4c)$$

$$G_4^4 = \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\ddot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{\alpha^2}{A^2}, \quad (4d)$$

$$G_1^4 = \alpha\left(\frac{\dot{B}}{B} - \frac{\dot{A}}{A}\right), \quad (4e)$$

B. VISCOUS FLUID:

The energy momentum tensor for a cloud of string dust with a bulk viscous fluid of string is given by

$$T_i^j = \rho u_i u^j - \lambda x_i x^j - \xi u^l;l\left(g_i^j + u_i u^j\right), \quad (5)$$

Where u_i and x_i satisfy the condition.

$$u^i u_i = -x^i x_i = -1, \qquad u^i x_i = 0, \quad (6)$$

In equation (5) ρ is the proper energy density for a cloud string with particle attached them, ξ is the coefficient of bulk viscosity, λ is the string tension density of particles, u^i is the cloud four-velocity vector and x^i is a unit space-like vector representing the direction of string. If the particle density of the configuration is denoted by ρ_p then we have

$$\rho = \rho_p + \lambda, \quad (7)$$

In a co-moving system of references such that ui = (0, 0, 0, 1), we have

$$T_1^1 = \xi\theta, \quad (8a)$$

$$T_2^2 = \xi\theta, \quad (8b)$$

$$T_3^3 = \lambda + \xi\theta, \quad (8c)$$

$$T_4^4 = \rho, \quad (8d)$$

$$T_1^4 = 0. \quad (8e)$$

C. FIELD EQUATION:

Einstein's field equations with 8πG = 1 and variable cosmological term Λ (t) in suitable units are

$$R_{ij} - \frac{1}{2}Rg_{ij} = T_{ij} - \Lambda(t)g, \quad (9)$$

The field equation (9) and (8) subsequently lead to the following system of equations:

$$\frac{\dot{B}}{B} + \frac{\dot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = \xi\theta - \Lambda, \quad (10a)$$

$$\frac{\dot{A}}{A} + \frac{\dot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = \xi\theta - \Lambda, \quad (10b)$$

$$\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = \lambda + \xi\theta - \Lambda, \quad (10c)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2} = \rho + \Lambda, \quad (10d)$$

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = 0, \quad (10e)$$

The scalar expansion θ and components of shear σ_{ij} are given by

International Journal of Engineering and Applied Sciences

www.eaas-journal.org

$$\theta = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}, \qquad (11)$$

$$\sigma_{11} = \frac{A^2}{3}\left[\frac{2\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right], \qquad (12)$$

$$\sigma_{22} = \frac{B^2 e^{-2\alpha x}}{3}\left[\frac{2\dot{B}}{B} - \frac{\dot{A}}{A} - \frac{\dot{C}}{C}\right], \qquad (13)$$

$$\sigma_{33} = \frac{C^2}{3}\left[\frac{2\dot{C}}{C} - \frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right], \qquad (14)$$

$$\sigma_{44} = 0, \qquad (15)$$

Therefore,

$$\sigma^2 = \frac{1}{3}\left[\frac{\dot{A}^2}{A} + \frac{\dot{B}^2}{B} + \frac{\dot{C}^2}{C} - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{C}\dot{A}}{CA}\right], \qquad (16)$$

3. SOLUTIONS OF THE FIELD EQUATION:

As the field equations (10) are a system of five equations with seven unknown parameters A, B, C, ρ, λ, ξ and Λ. Thus, two more relations are needed to solve the system completely. We assume that one condition is that the expansion scalar is proportional to the shear scalar θ ∝ σ, which leads to the

$$B = C^n, \qquad (17)$$

Where n is a constant and the second condition is

$$\Lambda = aH, \qquad (18)$$

Where H is Hubble parameter and a is a positive constant.

From equation (10e), we have

$$A = \gamma B, \qquad (19)$$

where γ is an integrating constant.

From equation (19), without any loss of generality we can take γ = 1.

$$A = B, \qquad (20)$$

In order to obtain the more general solution, we assume that the coefficient of bulk viscosity ξ is inversely proportional to the expansion θ, therefore we have

$$\xi\theta = K, \qquad (21)$$

Substituting equation (17) into equation (11), we have

$$\theta = (2n+1)\frac{\dot{C}}{C}, \qquad (22)$$

By the use of the equation (17), (19), (20) and (21), the field equation (10a) reduces to

$$\frac{\ddot{C}}{C} + n(n+1)\frac{\dot{C}^2}{C^2} = K - \frac{a}{3}(2n+1)\frac{\dot{C}}{C}, \qquad (23)$$

Equation (22) can be rewritten as

$$\frac{\ddot{C}}{C}/\frac{\dot{C}}{C} - \left(\frac{K}{n+1}\right)\frac{C}{\dot{C}} + \left(\frac{n^2}{n+1}\right)\frac{\dot{C}}{C} = -\frac{a}{3}\left(\frac{2n+1}{n+1}\right), \qquad (24)$$

On integrating equation (23), we obtain

ISSN2305-8269
International Journal of Engineering and Applied Sciences

www.eaas-journal.org

$$\dot{C} = C_1 C^{-\left(\frac{K+n^2}{n+1}\right)} e^{-\frac{a}{3}\left(\frac{2n+1}{n+1}\right)t}, \qquad (25)$$

Where C_1 is constant of integration.

Again integrating equation (24), we get

$$C = \left[\frac{K+n^2+n+1}{n+1}\right]^{\frac{n+1}{K+n^2+n+1}} \square \left[-\frac{3}{a}\left(\frac{n+1}{2n+1}\right)e^{-\frac{a}{3}\left(\frac{2n+1}{n+1}\right)t} + C_2\right]^{\frac{n+1}{K+n^2+n+1}}, \qquad (26)$$

Where C_2 is constant of integration

Therefore, from equation (17), (19) and (25), we obtain

$$B = \left[\frac{K+n^2+n+1}{n+1}\right]^{n\left(\frac{n+1}{K+n^2+n+1}\right)} \square \left[-\frac{3}{a}\left(\frac{n+1}{2n+1}\right)e^{-\frac{a}{3}\left(\frac{2n+1}{n+1}\right)t} + C_2\right]^{n\left(\frac{n+1}{K+n^2+n+1}\right)}, \qquad (27)$$

$$A = \gamma\left[\frac{K+n^2+n+1}{n+1}\right]^{n\left(\frac{n+1}{K+n^2+n+1}\right)} \square \left[-\frac{3}{a}\left(\frac{n+1}{2n+1}\right)e^{-\frac{a}{3}\left(\frac{2n+1}{n+1}\right)t} + C_2\right]^{n\left(\frac{n+1}{K+n^2+n+1}\right)}, \qquad (28)$$

Hence, the metric (1) reduces to form

$$ds^2 = -dt^2 + \left[\frac{K+n^2+n+1}{n+1}\right]^{\frac{2n(n+1)}{(K+n^2+n+1)}} \square \left[-\frac{3}{a}\left(\frac{n+1}{2n+1}\right)C_1 e^{-\frac{a}{3}\left(\frac{2n+1}{n+1}\right)t} + C_2\right]^{\frac{2n(n+1)}{(K+n^2+n+1)}} \square \left(\gamma^2 dx^2 + e^{-2\alpha x} dy^2\right) + \left[\frac{K+n^2+n+1}{n+1}\right]^{\frac{2(n+1)}{(K+n^2+n+1)}} \square \left[-\frac{3}{a}\left(\frac{n+1}{2n+1}\right)C_1 e^{-\frac{a}{3}\left(\frac{2n+1}{n+1}\right)t} + C_2\right]^{\frac{2(n+1)}{(K+n^2+n+1)}} dz^2, \qquad (29)$$

For the model (28), the expressions for the energy density ρ, the string tension density λ, the particle density ρ_p, the expansion scalar θ, the shear scalar σ and the cosmological term are, respectively given by

$$\rho = \left(n^2+2n\right)C_1^2\left[\frac{K+n^2+n+1}{n+1}\right]^{-2} \square \left[-\frac{3}{a}\left(\frac{n+1}{2n+1}\right)C_1 e^{-\frac{a}{3}\left(\frac{2n+1}{n+1}\right)t} + C_2\right]^{-2} e^{-\frac{a}{3}\left(\frac{2n+1}{n+1}\right)t} - \alpha^2\left[\frac{K+n^2+n+1}{n+1}\right]^{-2n\left(\frac{n+1}{K+n^2+n+1}\right)} \square \left[-\frac{3}{a}\left(\frac{n+1}{2n+1}\right)C_1 e^{-\frac{a}{3}\left(\frac{2n+1}{n+1}\right)t} + C_2\right]^{-2n\left(\frac{n+1}{K+n^2+n+1}\right)} - \frac{a(2n+1)}{3}C_1\left[\frac{K+n^2+n+1}{n+1}\right]^{-1} \square \left[-\frac{3}{a}\left(\frac{n+1}{2n+1}\right)C_1 e^{-\frac{a}{3}\left(\frac{2n+1}{n+1}\right)t} + C_2\right]^{-1} e^{-\frac{a}{3}\left(\frac{2n+1}{n+1}\right)t}, \qquad (30)$$

International Journal of Engineering and Applied Sciences

www.eaas-journal.org

$$\lambda=\left[\frac{-2n^2a+5na+a}{3(n+1)}\right]C_1\left[\frac{K+n^2+n+1}{n+1}\right]^{-1}\times\left[-\frac{3}{a}\left(\frac{n+1}{2n+1}\right)C_1e^{-\frac{a}{3}\left(\frac{2n+1}{n+1}\right)t}+C_2\right]^{-1}-\left(\frac{n^3+n^2+2n}{n+1}\right)C_1^2\left[\frac{K+n^2+n+1}{n+1}\right]^{-2}\times\left[-\frac{3}{a}\left(\frac{n+1}{2n+1}\right)C_1e^{-\frac{a}{3}\left(\frac{2n+1}{n+1}\right)t}+C_2\right]^{-2}-\alpha^2\left[\frac{K+n^2+n+1}{n+1}\right]^{-2n\left(\frac{n+1}{K+n^2+n+1}\right)}\times\left[-\frac{3}{a}\left(\frac{n+1}{2n+1}\right)C_1e^{-\frac{a}{3}\left(\frac{2n+1}{n+1}\right)t}+C_2\right]^{-2n\left(\frac{n+1}{K+n^2+n+1}\right)}-\left(\frac{n-1}{n+1}\right)K, \tag{31}$$

$$\rho_p=\left(\frac{2n^3+4n^2+4n}{n+1}\right)C_1^2\left[\frac{K+n^2+n+1}{n+1}\right]^{-2}\times\left[-\frac{3}{a}\left(\frac{n+1}{2n+1}\right)C_1e^{-\frac{a}{3}\left(\frac{2n+1}{n+1}\right)t}+C_2\right]^{-2}e^{-\frac{a}{3}\left(\frac{2n+1}{n+1}\right)t}-\left[\frac{8na+2a}{3(n+1)}\right]C_1\left[\frac{K+n^2+n+1}{n+1}\right]^{-1}\times\left[-\frac{3}{a}\left(\frac{n+1}{2n+1}\right)C_1e^{-\frac{a}{3}\left(\frac{2n+1}{n+1}\right)t}+C_2\right]^{-1}-\left(\frac{n-1}{n+1}\right)K, \tag{32}$$

$$\theta=(2n+1)C_1\left[\frac{K+n^2+n+1}{n+1}\right]^{-1}\times\left[-\frac{3}{a}\left(\frac{n+1}{2n+1}\right)C_1e^{-\frac{a}{3}\left(\frac{2n+1}{n+1}\right)t}+C_2\right]^{-1}e^{-\frac{a}{3}\left(\frac{2n+1}{n+1}\right)t}, \tag{33}$$

$$\Lambda=\frac{a(2n+1)}{3}C_1\left[\frac{K+n^2+N+1}{n+1}\right]^{-1}\times\left[-\frac{3}{a}\left(\frac{n+1}{2n+1}\right)C_1e^{-\frac{a}{3}\left(\frac{2n+1}{n+1}\right)t}+C_2\right]e^{-\frac{a}{3}\left(\frac{2n+1}{n+1}\right)t}, \tag{34}$$

$$\sigma=\frac{1}{\sqrt{3}}(n-1)C_1\left[\frac{K+n^2+n+1}{n+1}\right]^{-1}\times\left[-\frac{3}{a}\left(\frac{n+1}{2n+1}\right)C_1e^{-\frac{a}{3}\left(\frac{2n+1}{n+1}\right)t}+C_2\right]^{-1}e^{-\frac{a}{3}\left(\frac{2n+1}{n+1}\right)t}, \tag{35}$$

In summery, we have presented a new class of Bianchi type-IIII string cosmological models with bulk viscosity and cosmological term Λ(t). To obtain an explicit solution, we adopt the conditions $\xi \propto \theta$, $\theta \propto \sigma$, and $\Lambda \propto H$. Thus, the cosmological model for a string cosmology with bulk viscosity and cosmological term is obtained. The model describes a shearing non-rotating continuously expanding universe with a big-bang start. The physical and geometrical aspects of the model are also discussed. In the absences of cosmological term Λ, the model reduces to the string model with bulk viscosity investigated by Bali and Pradhan (2007).

References

[1] Misner C.W. (1967) : Nature. 214, 40.

[2] Misner C.W. (1968) : J. Astrophys. 151, 431.

[3] Nightingale J.P. (1973) : J. Astrophys. 185, 105.

[4] Heller M. and Klimek M. (1975) : Astrophys. Space Sci. 33, 261.

[5] Pavon D., Bafaluy J.and Jou D. (1991) : Class Quantum Gravity. 8, 357.

[6] Maartens R. (1995) : Class Quantum Gravity. 12, 455.

[7] Kalyani D. and Singh G.P. (1997) : In new direction in Relativity and Cosmology.

[8] Singh T., Beesham A. and Mbokazi W.S. (1998) : Gen. Rela. Gravit. 30, 537.
[9] Ng, Y. J. (1992) : Int. J. Mod. Phys. D 1, 145.
[10] Weinberg, S. (1989) : Rev. Mod. Phys. 61 1.
[11] Zeldovich, Ya. B. (1968) :Sov. Phys. Usp. 11, 381.
[12] Linde, A. D. (1974) : ZETP. Lett. 19, 183.
[13] Krauss, L. M. and Turner, M. S. (1995) : Gen. Rel. Grav. 27, 1137.
[14] A. G. Reiss et al. (2004) : Astron. J. 607 665.
[15] Singh, J. P., Tiwari, R. K.and Pratibha Shukla, (2007) : Chin. Phys. Lett. (12) 24.