SLIDING MODE CONTROL OF A MAGNETIC LEVITATION SYSTEM

B. M. MUSTAPHA AND J. D. JIYA

1Senior Lecturer, Department of Electrical and Electronic Engineering, University of Maduguri, Borno State Nigeria.  
2Prof., Electrical and Electronic Engineering Programme Abubakar Tafawa-Balewa University, Bauchi, Bauchi state, Nigeria.  
E-mail: bgmustaphaee@yahoo.com; jibjiya@yahoo.co.uk

ABSTRACT

Magnetic levitation system has posed significant challenges to control Engineers, because of the nonlinearities associated with the electromechanical dynamics. A variable structure controller is employed for controlling the open loop unstable system. The proposed controller has satisfactorily tracked the error to the desired value within the prescribed boundary layer. Simulation results reveal the effectiveness of the proposed controller.

Keywords: Maglev, Variable Structure System, Sliding Mode Controller, Sliding Surface, MATLAB

1 INTRODUCTION

Magnetic Levitation (Maglev) systems are widely used in various fields, such as frictionless bearings, high speed maglev passenger train, levitation of wind tunnel models and many others. The maglev systems are open loop unstable described by highly nonlinear differential equations, which present additional difficulties in controlling the system. The challenge in a magnetically levitated system is that of stabilization in such that the air-gap is kept constant by controlling the current in the coil. Therefore, it is an important task to construct high-performance feedback controller for regulating the position of the levitated object. In recent years, a lot of works have been reported in the literature for controlling magnetic levitation systems by taking nonlinearities of the system into account (Gultérez and Ro, 1988; Oleksiy et al., 2002; Shen, 2002; Walter and John, 1996; Yang et al., 2004).

Sliding mode control is a particular type of Variable Structure System (VSS) (Itkis, 1979; Utkin, 1977, 1978, 1983) that is designed to drive and then constrain the system to lie within a neighborhood of the switching function (Itkis, 1979). There are two main advantages of this approach. Firstly, the dynamic behavior of the system may be tailored by the particular choice of switching function. Secondly, the closed loop response becomes totally insensitive to particular class of uncertainty (Utkin, 1978). That is, the characteristic feature of a VSS is that the sliding mode occurs on a prescribed switching surface. The surface is the intersection of a set of discontinuity surface in the state space of a multiple state system, where each control input switches between two functions. This discontinuous surface are selected so that, while in sliding mode, the system performance satisfies the design objectives such as stability, chattering reduction, order reduction, etc. (Drazenvic, 1969; Wu-chung su et al., 1996).

This paper presents a sliding mode controller design for position control of magnetic levitation system. In order to compare the proposed controller with the other methods the performance of the sliding mode controller will be compared with that of PID controller. A sliding mode controller was designed, which controlled the system and the robustness of the closed loop system was discussed and simulation results are presented.

2 MAGNETIC LEVITATION DYNAMIC MODEL ANALYSIS

In maglev system model (Hung and Lin, 1995; Walter and Chiasson, 1996; Boudali et al, 2003; Kaloust et al, 2004; Mahdi and Farzan, 2009), the instantaneous flux linkage between the two magnetized bodies through the air gap is \( \phi_1(t) = \phi_1(t) \), then the inductance is given by:
\[ L(z) = \frac{\mu_o N^2 A}{2z(t)} \]  \hspace{1cm} (1)

\( \mu_o \) is the permeability, \( N \) is the number of turns and \( A \) is the pole area. Then the instantaneous voltage is also given by:

\[ v(t) = Ri(t) + \frac{\mu_o N^2 di(t)}{2z(t)} - \frac{\mu_o N^2 i(t) di(t)}{2z(t)^2} \frac{dt}{dt} \]  \hspace{1cm} (2)

The inductance of the electromagnet is varying with respect the levitation height \( z(t) \) as shown is the second and third terms of equation (2). The electromagnetic force produced by the current \( i(t) \) is given as:

\[ F(z,t) = \frac{i^2(t)}{2} \frac{dL(z)}{dz} \]  \hspace{1cm} (3)

The position \( z(t) \) of the levitating object will influence the inductance of the electromagnet coil, and the change are non linear and the balance between the electromagnetic force and the gravity is inherently unstable. And it is necessary to cancel gravitational force with the force produced by the electromagnet. Then the vertical dynamics of the system will be given as:

\[ m\ddot{z}(t) = -F(i,z) + mg - f_o(t) \]  \hspace{1cm} (4)

By equation (5) the open loop poles \( p_1 \) and \( p_2 \) changes with mass (m). However, pole \( p_3 \) formed by coil resistance and inductance will remains unchanged despite changes in mass and can be neglected. And the new transfer function will be

\[ G'(s) = \frac{-jk_1}{(ms^2 - k_2)} \]  \hspace{1cm} (6)

3 SLIDING MODE CONTROLLER DESIGN

Considering the reduced second order levitation system dynamics obtained from the transfer function in equation (6) the new state equation for the plant dynamics becomes:

\[ \dot{x}_1 = x_2 \]  \hspace{1cm} (7)

\[ \dot{x}_2 = \frac{k_2}{m} x_1 - \frac{jk_1}{m} u \]

Now, the second order equation is considered in designing a sliding mode controller that will achieve the robust stability of the levitation of maglev subsystem against mass variation and external disturbance.

A sliding mode controller is a variable structure controller. Basically a variable structure controller system employs structural change in order to exploit the useful properties derivable from a given set of structures of the system or even make a stable structure from combination of two unstable structures. The switching among different functions is determined by plant that is represented by a switching function. Consider designing a sliding mode controller for the maglev system in equation (7). And the following is a possible choice of the structure of a sliding mode controller.

\[ u = \phi^T x \]  \hspace{1cm} (8)

Where \( \phi^T = [\phi_1 \phi_2] \) is the vector of feedback gains which switch between a fixed set of values in accordance with a given logic on a switching plane defined by:

\[ \phi = \begin{cases} 
\alpha, & \text{if } sx_1 > 0 \\
\beta, & \text{if } sx_1 < 0 
\end{cases} \]  \hspace{1cm} (9)

\( S \) is called the switching function because the control action switches its sign on the two sides of the switching surface \( s = 0 \). \( s \) is defined as

\[ s = c^T x = 0 \]  \hspace{1cm} (10)

Where \( c^T = [c_1 \ c_2] \) and \( c_1 > 0 \) is a constant which is selected to prescribe the speed of response. The problem of variable structure controller design is that of finding the values of feedback vector \( \phi \) which guarantee sliding along the switching line.
defined by equation (10). The condition for sliding is given by (Utkin and Guldner, 1999; Utkin, 1977; 1979; 1983).

\[ s_\delta \leq 0 \] … (11)

3 LEVITATION SIMULATION RESULT

The results of simulation runs are presented to verify the operation of the levitation mathematical model and the performance of its controller using the MATLAB\SIMULINK software package. The simulation results show the uncontrolled and the controlled results.

Table 1 System parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X_c )</td>
<td>0.03</td>
<td>m</td>
</tr>
<tr>
<td>m</td>
<td>0.225</td>
<td>kg</td>
</tr>
<tr>
<td>R</td>
<td>2.48</td>
<td>Ohms</td>
</tr>
<tr>
<td>L</td>
<td>0.1793</td>
<td>H</td>
</tr>
<tr>
<td>( \beta )</td>
<td>200</td>
<td>V/m</td>
</tr>
<tr>
<td>I_o</td>
<td>5</td>
<td>A</td>
</tr>
<tr>
<td>C</td>
<td>7.938 \times 10^5</td>
<td>Nm^2/A^2</td>
</tr>
<tr>
<td>( k_1 )</td>
<td>5</td>
<td>N/A</td>
</tr>
<tr>
<td>( k_2 )</td>
<td>0.882</td>
<td>N/m</td>
</tr>
<tr>
<td>147</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Sources Ying Shing (2001)

3.1 Simulation results for uncontrolled system

The simulation plots in Fig 2 shows the responses of levitation height when no control algorithm is used. The system started to levitate after 3 sec showing uncontrolled levitation system.

3.2 Simulation results for controlled system

The control problem in variable structure control design can be stated as to force the system to track a desired time dependent trajectory in the state space using control signals. Such that the state will be brought from any initial point in the phase plane to the sliding surface, along which it slides toward the origin as shown in Fig 3. In Fig 4 and Fig 5 the tracking error was observed, this shows that the controller can track the levitation height and the vertical velocity to the desired values. In Fig 6 and 7 the sliding surfaces are shown and the system is guaranteed to be driven towards zero, remaining inside the prescribed boundary layers. Similarly in Fig 8 the control signal was also tracked to zero.

4 CONCLUSIONS

Magnetic Levitation system is open loop unstable system, the control problem is quite challenging because of the non-linearities associated with the electromechanical dynamics. A variable structure controller was proposed for control of magnetic levitation system. The proposed variable structure controller tracked the error to zero; this shows that levitation height can be tracked to any desired value. The simulation results also shows that the representative point will be brought from any initial point in the phase plane to the sliding surface and along which it slide towards the origin. The simulation results showed the effectiveness of the variable structure controller.

5 REFERENCES

3. Shen J.-C (2002), Non linear H\(^\infty\) control of magnetic levitation systems, Asian journal of control,4(3)pp1-24
10. Yuan Yih Hsu and Wah-Chun Chan (1984), "Optimal variable structure

Fig 2 Time response

Fig 3 Phase portrait (with initial condition 0.2)

Fig 4 Tracking error of levitation height

Fig 5 Sliding surface

Fig 6 Sliding surface

Fig 7 Control signal