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# FORMULATION OF 'PREDICTOR-CORRECTOR' METHODS FROM 2-STEP HYBRID ADAMS METHODS FOR THE SOLUTION OF INITIAL VALUE PROBLEMS OF ORDINARY DIFFERENTIAL EQUATIONS

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## ABSTRACT

*This study focuses on the formulation of Predictor-Corrector Methods from 2-Step Hybrid Adams Methods for the solution of Initial Value Problems (IVP) of Ordinary Differential Equations (ODE). The methods are derived on both grid and off-grid points using the Multistep Collocations Schemes which are evaluated at some points to produce Block Adams Moulton and Bashforth methods respectively. The methods with the highest order were selected from Hybrid Adams Bashforth and Moulton to serve as the Predictor-Corrector pair respectively. The convergence analyses unveil that the derived methods are valid and efficient. The numerical experiments were carried out (on stiff and non-stiff IVP) and reveal that Hybrid Adams methods performed better than the conventional Adams methods.*

**Keywords:** ***Initial Value Problem, Linear Multistep Method, Predictor-Corrector, Ordinary Differential Equations, Multistep Collocations Scheme.***

## 1. INTRODUCTION

Consider the numerical solution of the first order ordinary differential equation of the form;

$$y' = f(x, y) \qquad (1)$$

With the initial condition $y(x_0) = y_0$, there are many techniques of analytical solution of (1), but still many problems arising from field work leading to such equations without analytical solutions or where they exist the problem may be too cumbersome. Therefore numerical techniques remain the better alternative methods [9].

Essentially, the goal of any numerical method is to approximate a solution to the problem so as close to the actual solution as possible. Thus, the order of accuracy of a numerical method is considered of great importance in the analysis of the basic properties of the method [10].

The block method also approximates the solutions of (1.1), it contains more than one method each with distinct convergence property. [6] observe the block methodas based on the idea of simultaneously producing a 'block' of approximations $y_{n+1}, y_{n+2}, \cdots, y_{n+N}$.

The Adams Bashforth formular which is an explicit class of Adams Moulton method has been modified by [8]. Many efforts have been made to improve the implicit class as observed by [14], in view of the success recorded, there is need to improve and modify the implicit class so as to maintain its advantage over the explicit class.

Linear Multistep Collocation Method (LMM) is one of the techniques used in constructing block methods. Recently, [12, 13] attempt to extend the idea of collocation method to generate Adams methods and a more recent work of [5] to Adams Moultons method, an implicit method to retain its advantage over many methods. An improvement or modification was also made by [7] to tackle "starting value problems and complication of programming".

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Multistep collocation methods are good algorithms for deriving Adams methods for solving ODE as suggested by [3] and added that there is a better algorithm, for some problems, no satisfactory algorithm has been found and for others, we need several so that we can choose amongst them, depending on its speed and accuracy.

This paper intends to derive the Block Hybrid 2-step Adams Bashforth Methods and a Block Hybrid 2- step Adams Moulton Methods from their continuous schemes respectively at both grid and off-grid points to obtain the new discrete schemes which can be used to solve stiff and non-stiff initial value problems of ordinary differential equations.

The analysis of the order, error constant and the region of absolute stability of the newly constructed block methods will be discussed. These schemes shall be used simultaneously on stiff and non-stiff problems to ascertain their efficiency.

## 2. DERIVATION OF METHODS

Using the general multistep collocation methods by [11], the following D-matrix of Hybrid Adams Moulton's is

$$D=\begin{bmatrix} 1 & x_{n+1} & x_{n+1}^2 & x_{n+1}^3 & x_{n+1}^4 & x_{n+1}^5 \\ 0 & 1 & 2x_n & 3x_n^2 & 4x_n^3 & 5x_n^4 \\ 0 & 1 & 2x_{n+\frac{1}{2}} & 3x_{n+\frac{1}{2}}^2 & 4x_{n+\frac{1}{2}}^3 & 5x_{n+\frac{1}{2}}^4 \\ 0 & 1 & 2x_{n+1} & 3x_{n+1}^2 & 4x_{n+1}^3 & 5x_{n+1}^4 \\ 0 & 1 & 2x_{n+\frac{3}{2}} & 3x_{n+\frac{3}{2}}^2 & 4x_{n+\frac{3}{2}}^3 & 5x_{n+\frac{3}{2}}^4 \\ 0 & 1 & 2x_{n+2} & 3x_{n+2}^2 & 4x_{n+2}^3 & 5x_{n+2}^4 \end{bmatrix} \begin{bmatrix} \alpha_{01} & h\beta_{01} & h\beta_{\frac{1}{2}1} & h\beta_{11} & h\beta_{\frac{3}{2}1} & h\beta_{21} \\ \alpha_{02} & h\beta_{02} & h\beta_{\frac{1}{2}2} & h\beta_{12} & h\beta_{\frac{3}{2}2} & h\beta_{22} \\ \alpha_{03} & h\beta_{03} & h\beta_{\frac{1}{2}3} & h\beta_{13} & h\beta_{\frac{3}{2}3} & h\beta_{23} \\ \alpha_{04} & h\beta_{04} & h\beta_{\frac{1}{2}4} & h\beta_{14} & h\beta_{\frac{3}{2}4} & h\beta_{24} \\ \alpha_{05} & h\beta_{05} & h\beta_{\frac{1}{2}5} & h\beta_{15} & h\beta_{\frac{3}{2}5} & h\beta_{25} \\ \alpha_{06} & h\beta_{06} & h\beta_{\frac{1}{2}6} & h\beta_{16} & h\beta_{\frac{3}{2}6} & h\beta_{26} \end{bmatrix} = I \qquad (2)$$

The above algorithsm gives rise to the following continuous interpolant

$$\bar{y}(x)=y_{n+1}+\left(\begin{array}{l} \frac{1}{180}\frac{(x_n+h)\left(24x_n^4+126x_n^3h+224x_n^2h_n^2+151x_nh^3+29h^4\right)}{h^4} \\ +\frac{1}{6}\frac{\left(6h^4+25x_nh^3+35x_n^2h_n^2+20x_n^3h+4x_n^4\right)x_n}{h^4} \\ -\frac{1}{12}\frac{\left(25h^3+70x_nh^2+60x_n^2h+16x_n^3\right)x_n^2}{h^4} \\ +\frac{1}{18}\frac{\left(35h^2+60x_nh+24x_n^2\right)x_n^3}{h^4}-\frac{1}{6}\frac{\left(5h+4x_n\right)x_n^4}{h^4}+\frac{2x_n^5}{h^4} \end{array}\right)f_n$$

$$+\left(\begin{array}{l} -\frac{1}{45}\frac{(x_n+h)\left(-31x_nh^3-149x_n^2h_n^2-111x_n^3h-24x_n^4+31h^4\right)}{h^4}- \\ \frac{4}{3}\frac{x_n\left(6h^3+13x_nh^2+9x_n^2h+2x_n^3\right)x_n}{h^4}+\frac{2}{3}\frac{\left(6h^3+26x_nh^2+27x_n^2h+8x_n^3\right)x_n^2}{h^4}- \\ \frac{4}{9}\frac{\left(13h^2+27x_nh+12x_n^2\right)x_n^3}{h^4}+\frac{1}{3}\frac{\left(9h+8x_n\right)x_n^4}{h^4}-\frac{8x_n^5}{15h^h} \end{array}\right)f_{n+\frac{1}{2}}$$

$$+\left(\begin{array}{l} -\frac{1}{15}\frac{(x_n+h)\left(47x_n^2+12x_n^4+48x_n^3h-2x_nh^3+2h^4\right)}{h^4} \\ +\frac{x_n\left(19x_nh^2+4x_n^3+16x_n^2h+6h^3\right)}{h^4}-\frac{\left(19x_nh^2+8x_n^3+24x_n^2h+3h^3\right)x_n^2}{h^4} \\ +\frac{1}{3}\frac{\left(19h^2+24x_n^2+48x_nh\right)x_n^3}{h^4}-\frac{4\left(x_n+h\right)x_n^4}{h^4}+\frac{4x_n^5}{5h^4} \end{array}\right)f_{n+1}$$

$$+\left(\begin{array}{l} -\frac{1}{45}\frac{(x_n+h)\left(-81x_n^3h-24x_n^4-59x_n^2h^2-x_nh^3+h^4\right)}{h^4} \\ -\frac{4}{3}\frac{x_n^2\left(7x_n^2h+2x_n^3+7x_nh^2+2h^3\right)}{h^4}+\frac{2}{3}\frac{\left(21x_n^2h+8x_n^3+14x_nh^2-x_nh^3+h^4\right)}{h^4} \\ -\frac{4}{3}\frac{\left(21x_nh+12x_n^n+7h^2\right)x_n^3}{h^4}+\frac{1}{3}\frac{\left(7h+8x_n\right)x_n^4}{h^4}-\frac{8x_n^5}{15h^4} \end{array}\right)f_{n+\frac{3}{2}}$$

$$+\left(\begin{array}{l} \frac{1}{180}\frac{(x_n+h)\left(-24x_n^4-66x_n^3h-44x_n^2h^2-x_nh^3+h^4\right)}{h^4} \\ +\frac{1}{6}\frac{\left(4x_n^3+2x_n^2h+11x_nh^2+3h^3\right)x_n^2}{h^4}-\frac{1}{12}\frac{\left(16x_n^3+36x_n^2h+22x_nh^2+3h^3\right)x_n^2}{h^4}+ \\ \frac{1}{18}\frac{\left(24x_n^2+36x_nh+11h^2\right)x_n^3}{h^4}-\frac{1}{6}\frac{\left(4x_n+3h\right)x_n^4}{h^4}+\frac{2x_n^5}{15h^2} \end{array}\right)f_{n+2} \qquad (3)$$

Evaluating (3) at

$$x=x_n,\ x=x_{n+\frac{1}{2}},\quad x=x_{n+1},\ x=x_{n+\frac{3}{2}},\quad \text{and } x=x_{n+2}$$

produces the Block Hybrid Adams Moulton methods as;

$$y_{n+1}=y_n+\frac{h}{180}\left[29f_n+124f_{n+\frac{1}{2}}+24f_{n+1}+4f_{n+\frac{3}{2}}-f_{n+2}\right] \qquad (4)$$

$$y_{n+1}=y_{n+\frac{1}{2}}+\frac{h}{1440}\left[11f_{n+2}-74f_{n+\frac{3}{2}}+456f_{n+1}+346f_{n+\frac{1}{2}}+19f_n\right] \qquad (5)$$

$$y_{n+1}=y_{n+\frac{3}{2}}+\frac{h}{1440}\left[19f_{n+2}-346f_{n+\frac{3}{2}}-456f_{n+1}+74f_{n+\frac{1}{2}}-11f_n\right] \qquad (6)$$

$$y_{n+2}=y_{n+1}+\frac{h}{180}\left[29f_{n+2}+124f_{n+\frac{3}{2}}+24f_{n+1}+4f_{n+\frac{1}{2}}-f_n\right] \qquad (7)$$

Each of the above has order 5 and error constant: -1/5760, 11/92160, 11/92160, and -1/5760 respectively.

## 3. REGION OF ABSOLUTE STABILITY

Using MATLAB, the region of absolute stability of the Block Hybrid Adams Moulton at k=2 is plotted as shown on Figure 1.

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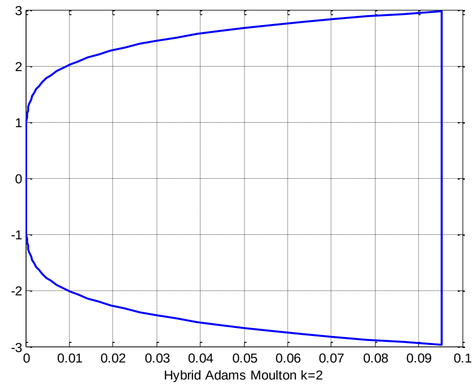


*Figure 1: Region of Absolute Stability of Hybrid Adams Moulton at k=2*

## 4. NUMERICAL EXPERIMENT

Two initial value problems (stiff and non-stiff) were solved using the Conventional Methods and the Newly Constructed Block Hybrid Methods for k=2 in order to test the efficiency of the derived methods.

Consider the stiff initial value problem below:

$$y(x) = -9y, y(0) = e, 0 \leq x \leq 1, h = 0.1$$

exact solution:

$$y(x) = e^{1-9x}, 0 \leq x \leq 1.$$

Solve the non-stiff initial value problem for the below:

$$y'(x) = -y, y(0) = 1, 0 \leq x \leq 1, h = 0.1$$

Exact solution:

$$y(x) = e^{-x}, 0 \leq x \leq 1$$

Solving the above equations numerically, by standard Adams methods and the New Block Hybrid methods respectively, we obtain the results shown in Tables 1-4 (some part in Appendix 1).

*Table 1: Numerical Results of Stiff Initial Value Problem*

| $x$ | Standard Adams-Methods for k=2 | Hybrid methods (with two off-grid points) for $k = 2$ | Exact solutions $y(x)$ |
|---|---|---|---|
| 0.1 | 1.116169023 | 1.101065187 | 1.116169023 |
| 0.2 | 0.469773633 | 0.447322350 | 0.449773633 |
| 0.3 | 0.196943367 | 0.181192054 | 0.182683524 |
| 0.4 | 0.082916632 | 0.073611677 | 0.074273578 |
| 0.5 | 0.034863568 | 0.029817091 | 0.030197383 |
| 0.6 | 0.014664854 | 0.012113589 | 0.012773390 |
| 0.7 | 0.006167789 | 0.007705509 | 0.004991593 |
| 0.8 | 0.002594167 | 0.003130465 | 0.002020294 |
| 0.9 | 0.001091092 | 0.001268024 | 0.0000825104 |
| 1.0 | 0.000458908 | 0.000515152 | 0.000335462 |

*Table 2: Numerical Result of Non-stiff Initial Value Problem*

| $x$ | Standard Adams-Methods for k=2 | Hybrid methods (with two off-grid points) for $k = 2$ | Exact solutions $y(x)$ |
|---|---|---|---|
| 0.1 | 0.9048374180 | 0.9048374180 | 0.904837418 |
| 0.2 | 0.8187349818 | 0.8187307492 | 0.818730753 |
| 0.3 | 0.7408258357 | 0.7408182137 | 0.740818220 |
| 0.4 | 0.6703303648 | 0.6703200365 | 0.670320046 |
| 0.5 | 0.6065430987 | 0.6065306482 | 0.606530659 |
| 0.6 | 0.5488256984 | 0.5488116230 | 0.548811636 |
| 0.7 | 0.4966005678 | 0.4965852895 | 0.496585303 |
| 0.8 | 0.44934050738 | 0.4493289490 | 0.449328964 |
| 0.9 | 0.4065863159 | 0.4065696441 | 0.406569659 |
| 1.0 | 0.3678963940 | 0.3678794252 | 0.367879441 |

## 5. ANALYSIS OF RESULT

A close observation of Tables 1, 2, 3 and 4 (see Appendix 1) reveals that both the derived methods (standard and hybrid Adams Methods) approximate the solutions of initial value problems given in examples (1) and (2) as their absolute errors are convergent.

In addition, the absolute errors presented in Tables 3 and 4 (Appendix 1) show that the discrete

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schemes of the newly constructed Block Hybrid methods with two off-grid points, perform better than the standard Adams methods when applied to stiff and non-stiff equations respectively. In other words, the Hybrid Adams Methods converge faster than the Standard Adams Methods, as they have minimum values of absolute error.

## 6. CONCLUSION

From the numerical experiments, it has been observed that the new methods perform better than the conventional one, because the error constant of the Hybrid methods are smaller than those of the conventional ones. Even though errors were large when the stiff problem was solved with new methods, but it is still minimum compared to that of the conventional one. Finally, we conclude that the new block method gives better results when applied to either stiff or non-stiff initial value problem.

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## APPENDIX 1

*Table 3: Absolute Errors of Stiff problem*

| $x$ | Exact solutions $y(x)$ | Standard Adams-Methods for k=2 | Hybrid Methods (with two off-grid points) for $k = 2$ |
|---|---|---|---|
| 0.1 | 1.116169023 | 1.10E-02 | 4.11E-03 |
| 0.2 | 0.449773633 | 2.04E-02 | 2.02E-03 |
| 0.3 | 0.182683524 | 1.43E-02 | 1.49E-03 |
| 0.4 | 0.074273578 | 8.64E-03 | 6.62E-04 |
| 0.5 | 0.030197383 | 4.67E-03 | 3.80E-04 |
| 0.6 | 0.012773390 | 1.89E-03 | 6.60E-04 |
| 0.7 | 0.004991593 | 1.18E-03 | 2.71E-03 |
| 0.8 | 0.002020294 | 5.74E-04 | 1.11E-03 |
| 0.9 | 0.0000825104 | 2.66E-04 | 4.43E-04 |
| 1.0 | 0.000335462 | 1.23E-04 | 1.80E-04 |

*Table 4: Table of Absolute Errors of Non-stiff Initial Value Problem*

| $x$ | Exact solution$y(x)$ | Standard Adams-Methods for k=2 | Hybrid Methods (with two off-grid points) for $k = 2$ |
|---|---|---|---|
| 0.1 | 0.904837418 | 3.60E-11 | 3.60E-11 |
| 0.2 | 0.818730753 | 4.22E-06 | 3.88E-09 |
| 0.3 | 0.740818220 | 7.61E-06 | 6.98E-09 |
| 0.4 | 0.670320046 | 1.03E-05 | 9.53E-09 |
| 0.5 | 0.606530659 | 1.24E-05 | 1.15E-08 |
| 0.6 | 0.548811636 | 1.41E-05 | 1.31E-08 |
| 0.7 | 0.496585303 | 1.52E-05 | 1.42E-08 |
| 0.8 | 0.449328964 | 1.15E-05 | 1.51E-08 |
| 0.9 | 0.406569659 | 1.66E-05 | 1.56E-08 |
| 1.0 | 0.367879441 | 1.69E-05 | 1.60E-08 |