June. 2014. Vol. 5. No. 01 ISSN2305-8269
International Journal of Engineering and Applied Sciences

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DEVELOPMENT AND ASSESSMENT OF NONLINEAR PREDICTORS FOR CONTINUATION METHOD

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Abstract:

The steps involved in the development of two nonlinear predictors for tracing the solution paths of power flow equations are presented in a clear and didactic way. Their performances are compared to those obtained by using the modified zero-order polynomial and the first-order polynomial predictors. Nonlinear predictors are based on the Lagrange interpolating polynomial of second degree, which allows reconstructing a function passing through three given points in the plane determined by the variables loading factor and voltage magnitudes, or voltage angles variables. The main advantage for using nonlinear predictors is obtaining a lower global number of iterations necessary to trace the whole solution curve. The proposed nonlinear predictors follow closely almost all the curvature of solutions trajectory, keeping a reduced number of iterations in corrector step comparing to linear predictors. Tests are performed for the IEEE-14, 30 and 300 bus systems, and for real large, heavy loaded 638-bus system.

Keywords: Nonlinear predictor; Continuation power flow; Parameterization technique; Lagrange interpolating polynomial.

1. INTRODUCTION

In the static voltage stability analysis, a set of nonlinear algebraic equations representing the electrical power system is used for computing the steady-state operating points for various loading and outage conditions [1- 2]. The two most widely used tools are the conventional power flow (PF) method and the continuation power flow (CPF) [3-10]. There is a growing interest in the power industry, even for small improvements of the PF and the CPF methods, to reduce the computational time required for tracing the P-V curve, without losing robustness [11-15]. The predictor step is used to estimate the next solution point. Depending on the curvature of the solution path and the predictor method choose, the estimation of the next solution will be more accurate, i.e., more close to the correct solution. When the prediction is an approximate solution, it becomes necessary to perform a correction to obtain the correct solution in order to avoid the accumulation of errors. The power flow using the Newton-Raphson method is the most used in corrector step and the value estimated by the predictor step will influence the convergence of the corrector. However, if the predictor solution is very close to the correct solution, it is considered as a good predictor and only a few iterations are needed to obtain the correct solution on the power flow solution trajectory (λ-V or P-V curves). Thus, the main feature desired for the predictor method is to have an estimation as close as possible to the correct solution, in order to reduce the computation time required for the tracing the whole P-V curve [13-15].

The objective of this paper is to present the steps involved in the development of nonlinear predictors in a clear and didactic way. The performance comparisons of a CPF considering proper nonlinear predictors to estimate the next solution is also presented. The nonlinear predictors are directly applied to P-V curve instead of the curves of variables (λ, V, or θ) in function of pseudo-arc length [12-16]. Two second-order based nonlinear predictors are presented. The results obtained by using the linear predictors are compared with the proposed nonlinear predictors to trace the P-V curves. The linear predictors are the trivial or modified zero-order polynomial, the secant or first-order polynomial, and the tangent. The nonlinear predictors are based on the Lagrange's interpolating polynomial of second-degree allowing reconstruct a function through three known operating points on the P-V curve. The improvements achieved with the proposed predictors occur along the whole P-V curve. The nonlinear predictors follow more closely the curvature of the solutions trajectory providing a lower global number of iterations in corrector step comparing with the linear predictors. In order to eliminate the singularity of the Jacobian matrix (**J**) at the MLP, and consequently the ill-conditioning and numerical problems, the voltage magnitude is used as the continuation parameter in the corrector step. This allows the computation of the MLP point with the desired precision, and also the complete tracing of the P–V curve. The proposed method is successfully applied to the IEEE-14, 30 and 300 bus systems and to the 638-bus system

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corresponding to part of South-Southeast Brazilian system. Comparisons are presented in terms of numbers of iterations and robustness.

2. CONTINUATION POWER FLOW

The Newton-Raphson method is an iterative technique used to solve the parameterized nonlinear algebraic equations of the power flow (PF) problem[6-10]. For each specified loading factor (λ), which is automatically changed, the values of voltage phase angle variables for all load (PQ) and generator (PV) buses, and the values of voltage magnitude variables for all load buses (PQ) are obtained from:

$$\begin{aligned}\Delta\mathbf{P}(\boldsymbol{\theta},\mathbf{V},\lambda)=\mathbf{P}^{sp}(\lambda)-\mathbf{P}(\boldsymbol{\theta},\mathbf{V})=\mathbf{0}\\ \Delta\mathbf{Q}(\boldsymbol{\theta},\mathbf{V},\lambda)=\mathbf{Q}^{sp}(\lambda)-\mathbf{Q}(\boldsymbol{\theta},\mathbf{V})=\mathbf{0}\end{aligned} \quad (1)$$

where the vector functions $\Delta\mathbf{P}(\boldsymbol{\theta},\mathbf{V},\lambda)$ and $\Delta\mathbf{Q}(\boldsymbol{\theta},\mathbf{V},\lambda)$ are the respective equations of real and reactive power balance of system buses, $\mathbf{V}$ is the vector of voltage magnitudes and $\boldsymbol{\theta}$ is the vector of voltage phase angles, $\mathbf{P}^{sp}(\lambda) = \lambda(\mathbf{P}_g^{sp} - \mathbf{P}_c^{sp})$ is the difference (mismatches) between the vectors of real power generation ($\mathbf{P}_g^{sp}$) and consumption ($\mathbf{P}_c^{sp}$) specified at PQ and PV buses, and $\mathbf{Q}^{sp}(\lambda) = \mathbf{Q}_g^{sp} - \lambda\mathbf{Q}_c^{sp}$ is the difference between the vectors of reactive power generated and consumed at PQ buses. In Refs. 3 and 4, an overview of the load flow problem and its solution using conventional methods can be found. In Ref. 4, two stochastic search methods, genetic algorithm and simulated annealing, are also presented. An MS-Excel Workbook, that allows students to evaluate the operation of the numerical methods without needing the advanced computational skills, is also included.

After obtaining the base case solution for λ equal to 1, the next operating points ($\boldsymbol{\theta}^j$, $\mathbf{V}^j$ e λ^j) are obtained considering successive increments in the λ value ($\lambda^{j+1} = \lambda^j + \Delta\lambda$). This procedure is repeated until the PF method becomes divergent or takes longer time for a solution, and the last converged solution is considered as the maximum loading point (MLP). The distance between the base case to the MLP is called static voltage stability margin. In Ref. 2 a comparison study that reveals the advantage and disadvantages of the commonly shunt devices, capacitor, SVC and STATCOM, used for static voltage stability margin enhancement is presented.

This procedure allows the computation of points very close to MLP, but not its accurate value due to the singularity of Jacobian matrix of the power flow equations at MLP. The value of MLP will also depend on the solution methodology and the step size used (Δλ). On the other hand, the LM can be accurately obtained by the continuation power flow, which consists of four basic elements: a predictor step, control step, parameterization procedure and the corrector step [5-12]. Observe that the above procedure using conventional power flow method is also a continuation method with the modified zero-order polynomial predictor (trivial predictor),[5-7] which use a current solution and a fixed increment in the parameter (λ) as an estimate for the next solution. Due to its robustness, the CPF is widely used nowadays and several parameterization techniques are employed to remove the Jacobian matrix singularity at the MLP of a P-V curve [6-12].

3. TYPES OF PREDICTION TECHNIQUES

Starting from the solution of (1) for the base case ($\boldsymbol{\theta}$, $\mathbf{V}$, λ=1), a predictor step is carried out to find an estimation for the next solution point. The predictor step has an important role in the CPF. Its main goal is to speed up the tracing of P-V curves allowing take larger step size around the MLP and providing more accurate predicted solutions via the reduced number of iterations which is necessary by the corrector step. Several different predictors have been proposed in the literature [5-6] and [13-15]. The most widely used predictors are the tangent and secant, and more recently, the nonlinear predictors (Lagrange's polynomial predictor) [13-15]. The Lagrange polynomial predictor consists of several orders; for a nonlinear predictor, the order must be at least two. This method is most used due to its simplicity, since it is not necessary to solve a system of simultaneous linear equations comparing with the tangent predictor [6], or to perform repetitive calculations as in the case of Newton interpolating polynomial. In this work, two second-order nonlinear predictors based on Lagrange's interpolating polynomial formula are used. The next sections describe the main characteristics of each predictor, including the proposed nonlinear predictor.

3.1. Tangent and Secant Predictor

The tangent predictor is a first order ODE (Ordinary Differential Equations) based method that uses the current solution and its derivatives to estimate the next solution. The tangent vector ($\mathbf{t}$) is obtained by taking the differential of eqn (1) expressed compactly as $\mathbf{G}(\boldsymbol{\theta}, \mathbf{V}, \lambda)=\mathbf{0}$, and by placing it in a matrix the following equation is obtained [6, 9-10]:

$$\begin{bmatrix}\mathbf{G}_\theta & \mathbf{G}_V & \mathbf{G}_\lambda\\ & \mathbf{e}_k & \end{bmatrix}\begin{bmatrix}d\boldsymbol{\theta}\\ d\mathbf{V}\\ d\lambda\end{bmatrix}\equiv\begin{bmatrix}\mathbf{G}_\theta & \mathbf{G}_V & \mathbf{G}_\lambda\\ & \mathbf{e}_k & \end{bmatrix}\mathbf{t}\equiv\mathbf{J}_m\mathbf{t}=\begin{bmatrix}\mathbf{0}\\ \mathbf{0}\\ \pm1\end{bmatrix} \quad (2)$$

where $\mathbf{G}_\theta = [(\partial\Delta\mathbf{P}/\partial\boldsymbol{\theta})^T\ (\partial\Delta\mathbf{Q}/\partial\boldsymbol{\theta})^T]^T$, $\mathbf{G}_V = [(\partial\Delta\mathbf{P}/\partial\mathbf{V})^T\ (\partial\Delta\mathbf{Q}/\partial\mathbf{V})^T]^T$ and $\mathbf{G}_\lambda = [(\mathbf{P}^{sp})^T\ (\mathbf{Q}^{sp})^T]^T$ are the partial derivatives of $\mathbf{G}$ with respect to $\boldsymbol{\theta}$, $\mathbf{V}$ and λ, respectively. $\mathbf{G}_\theta$ and $\mathbf{G}_V$ compose the Jacobian matrix $\mathbf{J}$ of the conventional power flow (PF).[3,4] The new column ($\mathbf{G}_\lambda$) added to $\mathbf{J}$ corresponds to the new variable λ. Since the number of unknown variables is larger than the number of equations, one of the elements of $\mathbf{t}$ must be set to a value differ from zero. This variable (V_k, θ_k or λ) is called as the continuation parameter. It is noteworthy that a parameter change is necessary to guarantee the nonsingularity of the augmented Jacobian matrix $\mathbf{J_m}$ in the MLP.

June. 2014. Vol. 5. No. 01 ISSN2305-8269
International Journal of Engineering and Applied Sciences

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In the local parameterization technique, the parameter changes from λ to the voltage magnitude of the bus whose magnitude has the largest rate of change near to the MLP [6-9]. So, $\mathbf{e_k}$ is a row vector appropriately dimensioned, with null elements, except the k'th, which is equal to 1. The number 1 is placed in the column of variable which was chosen as the continuation parameter. Letting $t_k = \pm 1$ imposes a non-zero norm on **t**. The choice of sign (±1) depends on how the variable chosen as parameter is varying, positive if it is increasing and negative, if it is decreasing. The estimate ($\boldsymbol{\theta}^e$, $\mathbf{V}^e$, λ^e) for the next solution is found by taking an appropriately sized step (σ) in the direction of the tangent to the curve at a current solution.

Unlike the tangent predictor which needs only one solution on the curve to estimate the next one, the secant predictor or first-order polynomial predictor is a polynomial extrapolation-based method, which uses the current ($\boldsymbol{\theta}^j$, $\mathbf{V}^j$, λ^j) and previous solutions ($\boldsymbol{\theta}^{j-1}$, $\mathbf{V}^{j-1}$, λ^{j-1}) to estimate the next one [5, 7]. So, to start first-order secant predictor it is necessary at least two solution points on the P-V curve. These points are obtained by a conventional PF or a CPF. The next solution point is obtained by taking an appropriately sized step (σ) in the direction of the straight line that passes through the two last solution points in the curve. A trivial and inexpensive predictor is the modified zero-order polynomial which uses the current solution and a fixed increment in the parameter as an estimate for the next one [5, 7].

3.2. The Lagrange's Polynomial Interpolation Formula

From Fig. 1 one can verify that the loading factor λ presents an almost quadratic relationship with respect to voltage magnitude, i.e. the P-V curve is similar in shape to a quadratic function. Based on this similarity, several papers proposed the use of quadratic nonlinear predictors based on quadratic or Lagrange's interpolating polynomial of second degree [13-15]. During the tracing of the P-V curve, the Lagrange interpolation is used as an extrapolation technique to estimate the next values of the variables (λ, **V**, or **θ**), which are beyond the known observation interval.

The Lagrange's interpolating polynomial is an at most n degree polynomial approximation formula to the function which passes through $n+1$ given points ((x_0, $y_0 = f(x_0)$), (x_1, $y_1 = f(x_1)$), …, (x_n, $y_n = f(x_n)$)). It can be expressed in the most compact representation by the Lagrange form:

$$P(x) = \sum_{k=0}^{n} L_k(x) f(x_k) \quad (3)$$

where $P(x)$ represents the Lagrange interpolating polynomial, x_k is the given discrete point ($k=0, 1, \ldots, n$), $f(x_k)$ is the function value at x_k, and $L_k(x)$ is the Lagrange's interpolating coefficient which is given by:

$$L_k(x) = \prod_{\substack{m=0 \\ m \neq k}}^{n} \frac{x - x_m}{x_k - x_m} \quad (4)$$

where n is the order of the approximating polynomial, and m is the degree count $0 \leq m \leq n$.

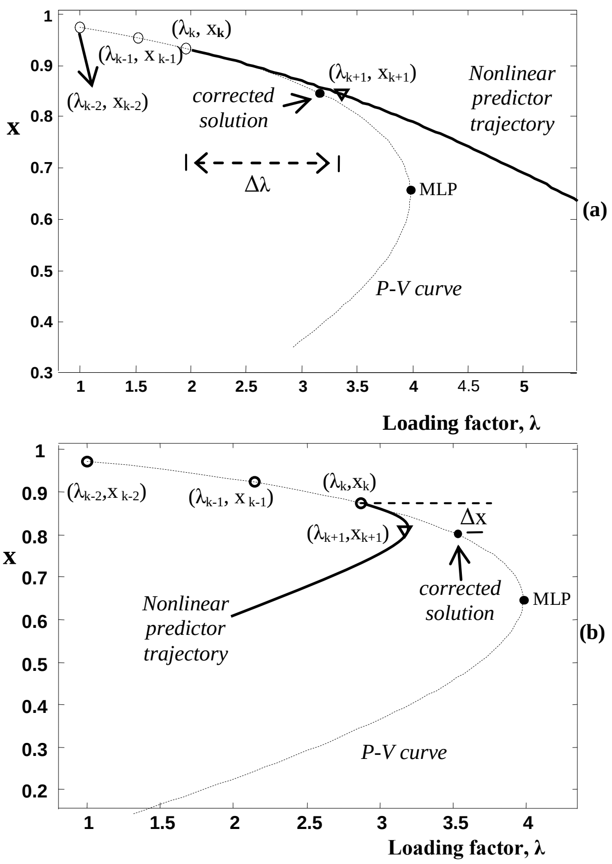

Fig. 1 Obtaining of nonlinear predictors considering different second-order approximation polynomials: (a) by using the nonlinear predictor of Eq. (5) and (b) by using the nonlinear predictor of Eq. (6).

3.2.1. Development of the nonlinear predictors

Due to the difficulties to obtain new solution of predicted points on the lower part of P-V curves, in Refs 13-15 the nonlinear predictors are applied to curves of variables (λ, **V**, or **θ**) in function of pseudo-arc length instead of directly in P-V curves. Nevertheless, it is noteworthy that in this work the nonlinear predictor is directly applied to P-V curve and two second-order based nonlinear predictors obtained by the quadratic Lagrange's interpolating polynomial formula are presented. The two second-order approximation polynomials passing through three known solution points (λ_{k-2}, $\mathbf{x_{k-2}}$), (λ_{k-1}, $\mathbf{x_{k-1}}$) and (λ_k, $\mathbf{x_k}$) are given by:

ISSN2305-8269
International Journal of Engineering and Applied Sciences

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$$x_{k+1}(\lambda)=\sum_{j=k-2}^{k}L_j(\lambda)x_j(\lambda_j)=\frac{(\lambda-\lambda_{k-1})(\lambda-\lambda_k)}{(\lambda_{k-2}-\lambda_{k-1})(\lambda_{k-2}-\lambda_k)}x_{k-2}+\frac{(\lambda-\lambda_{k-2})(\lambda-\lambda_k)}{(\lambda_{k-1}-\lambda_{k-2})(\lambda_{k-1}-\lambda_{k-2})}x_{k-1}+\frac{(\lambda-\lambda_{k-2})(\lambda-\lambda_{k-1})}{(\lambda_k-\lambda_{k-2})(\lambda_k-\lambda_{k-2})}x_k \tag{5}$$

$$\lambda_{k+1}(x)=\sum_{j=k-2}^{k}L_j(x)\lambda_j(x_j)=\frac{(x-x_{k-1})(x-x_k)}{(x_{k-2}-x_{k-1})(x_{k-2}-x_k)}\lambda_{k-2}+\frac{(x-x_{k-2})(x-x_k)}{(x_{k-1}-x_{k-2})(x_{k-1}-x_{k-2})}\lambda_{k-1}+\frac{(x-x_{k-2})(x-x_{k-1})}{(x_k-x_{k-2})(x_k-x_{k-2})}\lambda_k \tag{6}$$

where x represents each one of the elements of the vector $\boldsymbol{\theta}$ or $\mathbf{V}$.

As depicted in Fig. 1, eqn (5) corresponds to a parabola that is opened downward (for solution points belonging to the upper part of P-V curve) or upward (for solution points belonging to the lower part of P-V curve) and eqn (6) to a parabolas that are opened leftward. In this works, both parabolas will be used as an extrapolation method to estimate the next values of the loading factor (λ) and the voltage magnitudes ($\mathbf{V}$) and phase angles ($\boldsymbol{\theta}$) variables belonging to the set of PQ buses, which includes the set of PV buses whose limits of reactive power were hit and their types were switched to PQ.

The adoption of this procedure is due to the almost quadratic relationship between λ and the voltage magnitudes or phase angles variables of these buses, i.e., the curves are similar in shape to a quadratic function and so, it provides a better performance of the proposed nonlinear predictors. Equation (5) is used after obtaining the first three solution points and after the nose point is passed. Equation (6) is used around the MLP where the power systems conditions present highly nonlinear characteristics. Once the Lagrange interpolating polynomial function was determined, the next solution $\mathbf{x}_{k+1}(\lambda_{k+1})$ is computed using eqn (5) and considering a step size in λ, $\lambda_{k+1}=\lambda_k+\Delta\lambda_k$. Near to the MLP the nonlinear predictor is changed and the next predicted solution $\lambda_{k+1}(\mathbf{x}_{k+1})$ is determined using eqn (6) and considering a step size in $\mathbf{x}_k$, $\mathbf{x}_{k+1}=\mathbf{x}_k+\Delta\mathbf{x}_k$.

Figure 2 illustrates the different solution points estimated by the two nonlinear predictors. As can be observed in Fig. 2(a), in case of eqn (5) the same value of λ is used to obtain the predicted solutions $\mathbf{x}_{k+1}$ ($\boldsymbol{\theta}$ or $\mathbf{V}$), i.e., all the predicted solutions are vertically aligned at λ_{k+1}. On the other hand as shown in Figs. 2(b) and (c), when eqn (6) is used, the same step size in each element of $\mathbf{x}_{k+1}$ (Δx_k) will result in different values for λ. So, intending to keep the same value for λ, one of variables of $\mathbf{x}_{k+1}$ is chosen as reference:

$$x_{ch}\leftarrow\max\left\{\left|\mathbf{V}^{j+1}-\mathbf{V}^{j}\right|,\left|\lambda^{j+1}-\lambda^{j}\right|\right\} \tag{7}$$

where j refers to the solution point on the curve. Equation (7) compares the voltage magnitudes of the buses and the λ of the current and previous solutions, the chosen variable is that of the vector $\mathbf{V}$ which presents the highest rate of change in its value, which is one of the characteristics of a critical bus [1]. Its corresponding voltage magnitude in vector $\mathbf{x}_{k+1}$ is adopted, ensuring a better approximation to the solution trajectory curves, see Figs. 3(a) and (b). So, after choosing the reference bus, the respective λ_{k+1} is computed by using the nonlinear predictor of eqn (6). Following, its value is used to estimate the value of each of elements of vector $\mathbf{x}_{k+1}$ by using the eqn (8):

$$x_{k+1}=\frac{f-\sqrt{f^2+4ed}}{2e},\ e\neq 0 \tag{8}$$

where

$$f=bc(x_k+x_{k-1})\lambda_{k-2}+ac(x_k+x_{k-2})\lambda_{k-1}+ab(x_{k-2}+x_{k-1})\lambda_k$$

$$e=bc\lambda_{k-2}+ac\lambda_{k-1}+ab\lambda_k\,;$$

$$d=abc\lambda_{k+1}-bcx_{k-1}x_k\lambda_{k-2}-acx_{k-2}x_k\lambda_{k-1}-abx_{k-2}x_{k-1}\lambda_k\,;$$

and

$$a=(x_k-x_{k-2})(x_{k-1}-x_{k-2});$$
$$b=(x_k-x_{k-1})(x_{k-2}-x_{k-1});$$
$$c=(x_{k-2}-x_k)(x_{k-1}-x_k);$$

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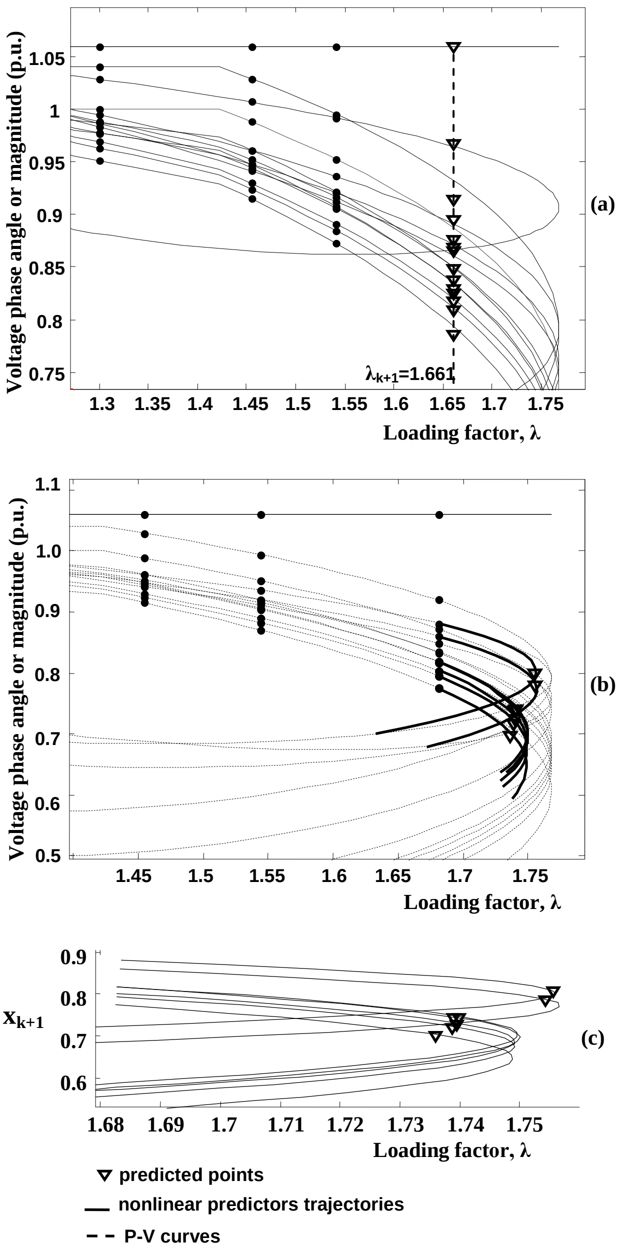

Fig. 2 Predicted solutions obtained: (a) by using the nonlinear predictor of Eq. (5) with a step size in λ, (b) by using the nonlinear predictor of Eq. (6) with a step size in x and (c) their respective trajectories and predicted solution points.

4. THE CORRECTOR STEP

The corrector step is necessary to correct the predicted solutions avoiding error accumulation. In this work the voltage magnitude of the bus that presents the highest rate of change in voltage magnitude is used as the continuation parameter in the corrector step to remove the singularity of matrix at MLP. After the prediction has been made, the equation y − ye = 0, where y and ye are, respectively, the variable selected as the continuation parameter and its predicted value, is appended to eqn (1), which is solved by a Newton method. It is worth mentioning that the choice of continuation parameter is

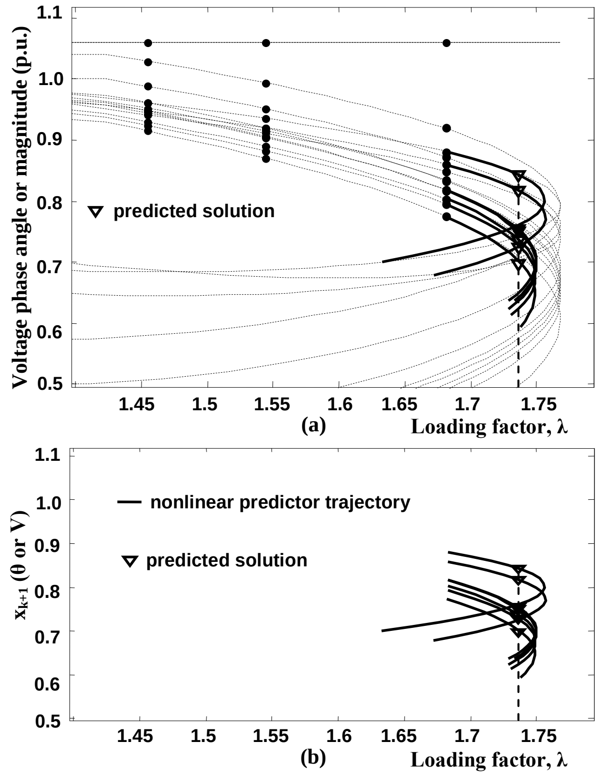

Fig. 3 (a) P-V curves and the nonlinear predictors obtained by using Eq. (6), (b) trajectories of the nonlinear predictor of Eq. (6) with predicted solution obtained by using Eq. (8).

crucial to ensure the methods do not fail to obtain the MLP, especially for power system with voltage instability problems that have the strong local characteristic. For systems like this, the P-V curve of most buses turning sharply with respect to λ. Both the loading factor and the voltage magnitude present a simultaneous reversion in its variation tendency, reaching their maximum value at the MLP. In other words, the Jacobian matrices are singular at the MLP. For these cases, the local parameterization technique is considered as the only way to eliminate the singularity, given that all other global parameterization techniques, or the arc length, or that uses a vector perpendicular to the tangent vector at the curve, fail in obtaining the MLP solution, as stated in Ref. 8. The most relevant difficulties that are present during the choice of continuation parameter are presented in detail in Ref. 12.

5. Test Models, Results and Discussion

For all simulations, the power mismatch convergence threshold was 10^{-4} p.u. The procedure of accounting for Q-limit at PV buses and tap limits in the on load-tap changing (OLTC) transformers is the same as in ordinary power flow method. In each iteration, the reactive generation of all PV buses is compared to their respective limits. In case of violation, a PV bus is switched to type PQ. This bus can be switched back to PV in the next iterations. Tap limit violations are also checked. The step size used for the tracing of the P-V curves was $\Delta\lambda = \Delta x = 0.15$. In tests performed, the first solution points required by each predictor (one for the tangent, two for the secant

June. 2014. Vol. 5. No. 01 ISSN2305-8269
International Journal of Engineering and Applied Sciences

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and three for the nonlinear predictors) are obtained by a conventional Newton-Raphson power flow [3-4].

5.1. An Illustrative Example of Nonlinear Predictor Performance

A simple 3-bus, 2-branch system is used to illustrate the performance of the nonlinear predictor, which system data is given in Table 1 and Table 2.

TABLE 1 Bus data for the 3 buses, 2 branch system.

Bus	Type	P (p.u.)	Q (p.u.)	V (p.u.)	θ (rad.)
1	Vθ	−	−	1.0	0.0
2	PV	-0.15	0.0	1.0	−
3	PQ	-0.05	-0.02	−	−

TABLE 2 Transmission lines data for the 3 buses, 2 branch system.

Transmission line		r (p.u.)	x (p.u.)	b_{k-m}^{sh} (p.u.)
from	to			
1	2	0.1	1.0	0.01
1	3	0.2	2.0	0.02
2	3	0.1	1.0	0.01

Figure 4 shows the curves of voltages and angles variables as a function of λ. These figures also show the predicted solutions computed via different predictors along with the number of iterations which is necessary by the corrector step to obtain each solution point of the curve, as shown in Fig. 4(c). In Fig. 4(b) a detail of region around of the MLP is shown. Note that, since the estimate values for each predictor could be different, in order to have a better comparison between the linear and nonlinear predictors, a procedure was adopted to guarantee that these values will take the system from the same initial point to the same final point. So, first the corresponding parameters of each state were obtained by using the nonlinear predictors in the predictor step and the voltage magnitude as the continuation parameter in the corrector step. Once all the states were obtained, the respective predictor step sizes (σ) were computed and after this, they were used to estimate the next solution point by respective linear predictors; here, the voltage magnitude was used as the continuation parameter in the corrector step. It is also shown in Fig. 4(b) the number of iterations required to obtain each solution point on the P-V curve. Note to the changes from the eqn (5) (a parabola in λ) to eqn (6) (a parabola in x, i.e., in **θ** or **V**) close to the MLP the nonlinear predictor. This exchange of variables is done according eqn (7). The aim of the adoption of this criterion for exchange of variables is to lead to an improvement performance of the algorithm, because it leads to an appropriate choice between parabolas. As it can be seen from Fig. 4(b), while λ presents the highest rate of change close to a given solution, the nonlinear predictor of eqn (5) is used. Otherwise, when one of voltage magnitudes presents highest rate of variation, the nonlinear predictor of eqn (6) is used.

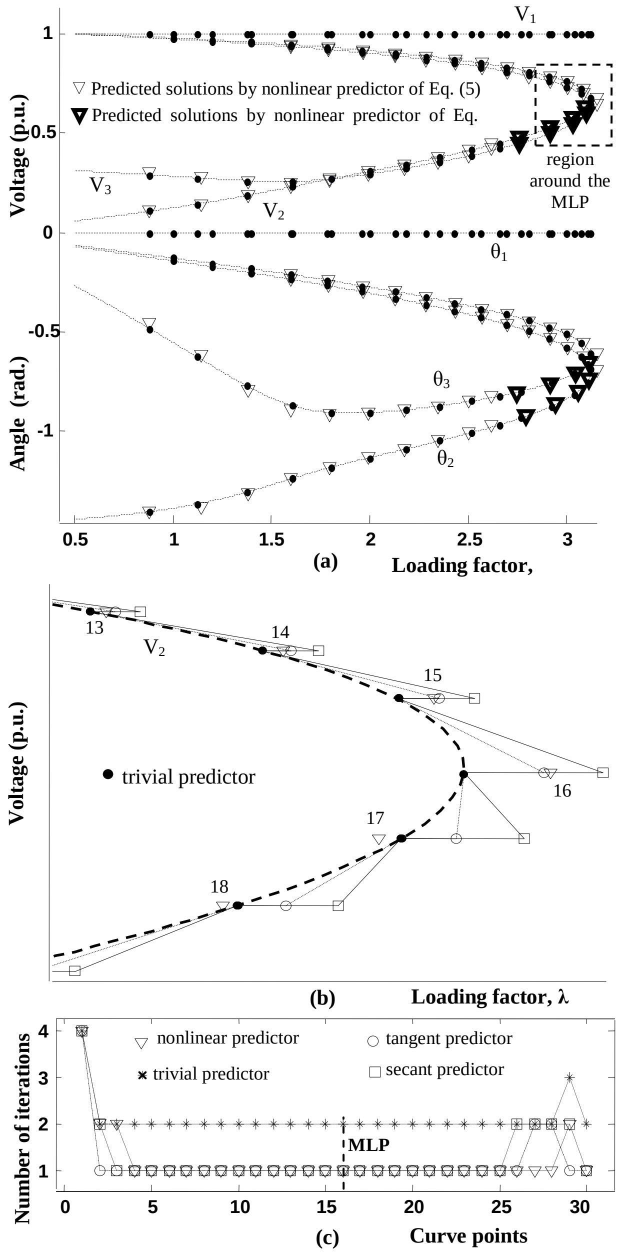

Fig. 4 (a) Performance of the nonlinear predictors for the 3-bus simple system, (b) detail of region around the MLP showing the predicted points obtained by using different predictors and (c) the respective number of iterations necessary by the corrector step.

Generally, for lightly and normal loaded conditions, λ presents the greatest variations while in heavier loading situations, close to the MLP, the voltage magnitudes show larger variations, as it happens from solution point 15 to

International Journal of Engineering and Applied Sciences

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16 which is shown in Fig. 4(b). Then, the next solution point 17 is computed by using eqns (6) and (7). After the change of the sign of λ, at least three solution points before switching the variable are still calculated; see Fig. 4(a). The trivial predictor needs additional iterations to converge than the other predictors, which in turn present practically the same performance. The secant and nonlinear predictors estimate the new solutions from polynomial equations. Therefore, these predictors need a little less execution time than the tangent predictor, which needs to use the most recent inverse of Jacobian matrix.

5.2. Performance of The Nonlinear Predictors for the IEEE Systems

Figure 5 shows the results obtained for the IEEE-14 bus system. In Fig. 5(a) the P-V curve of the critical bus (bus 14) with the solution points computed by the nonlinear predictors is presented. In this figure a detail consideration of the region around of the MLP shows the

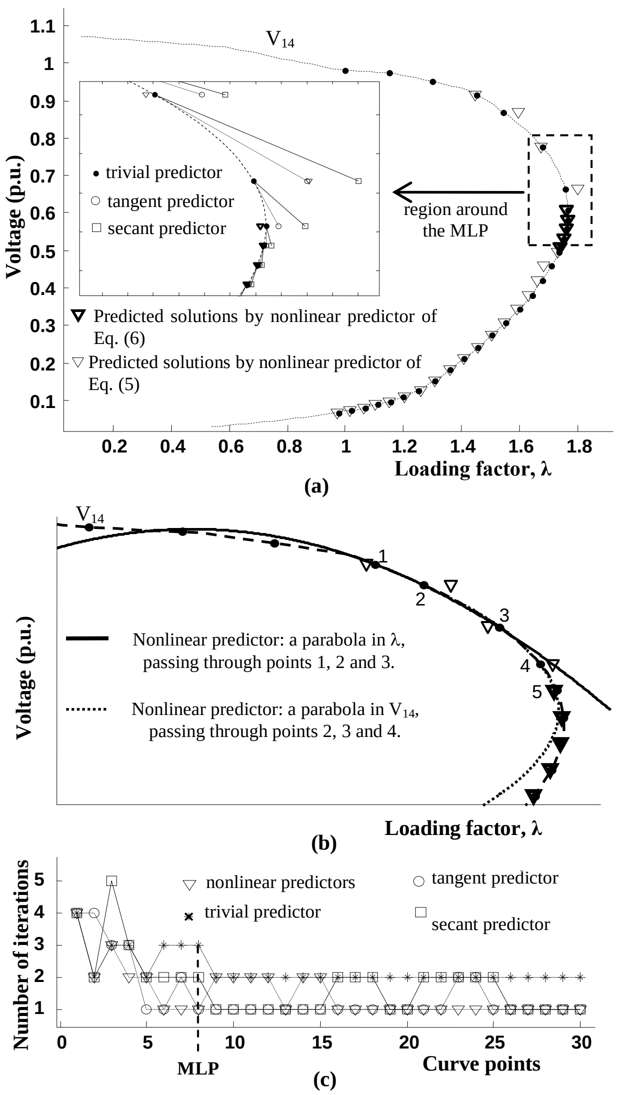

Fig. 5 Performance of the nonlinear predictors for the IEEE-14 bus system (a) P-V curve with the predicted points, (b) parabolas of the nonlinear predictors and (c) the number of iterations needed to obtain each point.

predicted solution points computed by each one of the linear (trivial, secant and tangent) and nonlinear predictors. Figure 5(b) shows details of the nonlinear predictor trajectories and Fig. 5(c) the number of iterations needed to obtain each solution point of the curve.

Figures 6(a) and 6(b) present for the IEEE-30 and IEEE-300 bus systems, the respective number of iterations needed to obtain each solution point of the P-V curve for the linear and nonlinear predictors. From this figure, it is worth mentioning that for the IEEE-300 the number of iterations of the nonlinear predictors were around two iterations. This occurs because some P-V curves of the set of PQ buses present a sharp nose, as previously mentioned. So when in the vicinity of MLP leads to a prediction of solution points, which lie a little bit far away from the curvature of solutions trajectory.

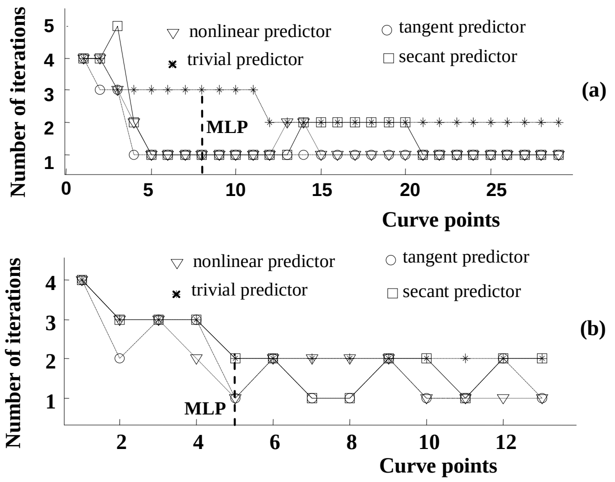

Fig. 6 Number of iterations needed to obtain each solution point by the corrector steps for the: (a) IEEE-30, (b) IEEE-300.

5.2.1. Performance of the nonlinear predictors for the 638-bus systems

In the assessment of predictor techniques, comparing their performance in the trace of the P-V curve of large systems is considered as an important factor. Hence, in Fig. 7 the performances of the predictor techniques for a large real, very stressed (heavily loaded) system corresponding to a part of South-Southeast Brazilian system, with 638-bus and 1276-branch, are compared.

In Figs. 7(a) and (b) the respective number of iterations and the computational times required to obtain each point in the P-V curve are shown. The CPU times presented in Fig. 7(b) were normalized by the respective CPU times required by the nonlinear predictors. For each predictor, Figs. 7(c) shows the overall normalized CPU time, and Fig. 7(d) shows the respective percentage of CPU time. The results show that it is possible to obtain a reduction in computational time, therefore, an efficiency improvement with the use of nonlinear predictors techniques is obtained.

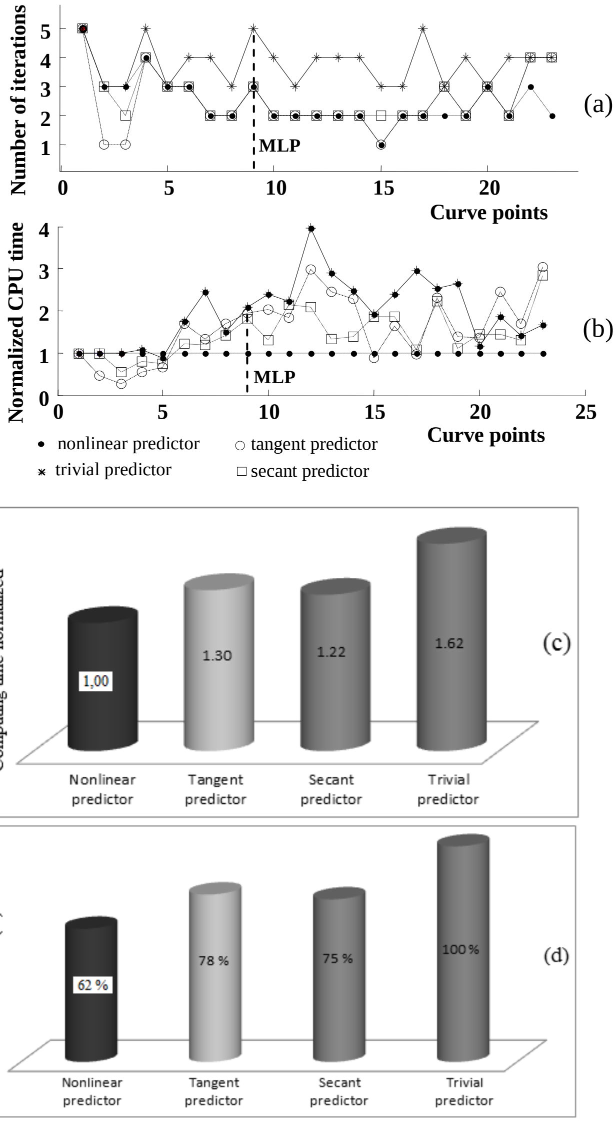

Fig. 7 Performance of the nonlinear and linear predictors for the 638-bus system: (a) number of iterations needed to obtain each solution point by the corrector steps, (b) normalized CPU time needed to obtain each solution point, (c) overall normalized CPU time for each predictor, (d) percentage of CPU time for each predictor.

5.3. Overall Performance of the Nonlinear Predictors

Figure 8 show the total number of iterations with the percentage of global number iterations which is necessary by each of the predictors for all the analyzed system. The total number of necessary iterations by the tangent and nonlinear predictors represents only 21% of total number of iterations. So, both predictors showed a good overall performance, but the tangent predictor needs a little more time, despite using the last inverse Jacobian matrix. From this figure it can be seen that among the predictors analyzed, the nonlinear predictor presents the best performance.

6. CONCLUSIONS

This paper presents the straightforward and easy to implement quadratic nonlinear predictors based on Lagrange Interpolating polynomial formula. The nonlinear predictors are directly applied to P-V curve and are used as an extrapolation technique to estimate the next solution point. Two functions are used as nonlinear predictors, one corresponds to a parabola in λ that is opened downward (for points belonging to the upper part of P-V curve) or upward (for points belonging to the lower part of P-V curve) and the other to a parabola in $\mathbf{V}$ (or $\boldsymbol{\theta}$) that is opened leftward. A criterion to switch from one nonlinear predictor to another, during the tracing of the P–V curves is also presented. The automatic switching procedure is done with the purpose to keep a low requirement in terms of the number of iterations by using a proper nonlinear predictor to estimate the next solution in each part of the curve. The voltage magnitude is used as the continuation parameter in the corrector step to avoid the singularity of the Jacobian matrix and to make a possible accurate determination of MLP.

A comparison between linear predictors (trivial, secant and tangent) and nonlinear predictors in terms of numbers of iterations, computing time and robustness is presented. Comparing with the tangent and secant predictors, the second-order based nonlinear predictors often presents less error to estimate the new solutions; i.e., predicted solutions points closer to the curvature of P-V curve are

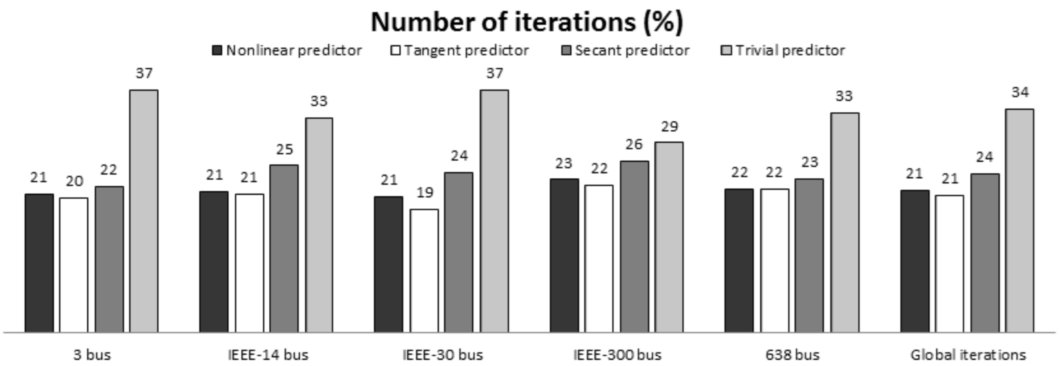

Fig. 8 Comparison for each system of the overall number of iterations needed to compute the P-V curve considering each predictors and global number of iterations.

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provided. An efficiency improvement in the P-V curve tracing is obtained with the use of nonlinear predictors. For large real power system, it was shown that the nonlinear predictor presents the best performance.

7. ACKNOWLEDGMENTS

Work was supported by CAPES and CNPq (Brazilian Research Funding Agencies).

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