Feb , 2014. Vol. 4, No. 8 ISSN2305-8269

International Journal of Engineering and Applied Sciences

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SPATIAL INTERPOLATION AND CALCULATION OF THE VOLUME AN IRREGULAR SOLID

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ABSTRACT

Into the modelling process are entering the most frequently isolated points with different density and distribution. To estimate of the values where are not available data are used various spatial interpolation methods. Choosing the optimal interpolation method is very subjective and can greatly affect the accuracy of the interpolated values. The present contribution is trying to estimate unmeasured values by spatial interpolation methods implemented in the application Spatial Analyst and by ABOS method applied on the measured data of heap. By such techniques will be created surface and will be used to calculate the cubic capacity modelled heap and obtained values will compare with each other.

Keywords: Spatial interpolation, IDW, ABOS

I. INTRODUCTION

In a geodetic practice, we often encounter with the task of measurement data on surface and then analyze those, or create a model of the measured surface. Existentially is not possible to cover observations, respectively measuring the whole area of interest. For this reason are for estimating values in areas without direct observation are using spatial interpolation methods. These interpolation methods are based on the principle of spatial autocorrelation – ie. on the assumption that the values predicted quantity at locations near you will be more similar than those in distant locations. Spatial interpolation methods are often included in many applications of programs. One of them is the application of Spatial Analyst, which provides the interpolation of several selected methods. An innovative method of interpolation introduced in 2009 M. Dressler called ABOS (Approximation (interpolation) Based On Smoothing), which uses basic mathematical functions - numeric tension and extermination.

The present article describes how to estimate the surface treatment of these interpolation methods in programs ArcGIS 9.x and SurGe 65.0. The resulting surface is used to calculate the cubic capacity heap and the volumes are compared with each other.

II. INTERPOLATION METHODS

Interpolation is procedure of the estimation of unknown values from known (measured) values in the area. It is based on the notion of approximate function $f(x)$, which replaces the original function $f_0(x)$ with some precision, so called ”gridding” consists in estimating the values of points distributed on a regular network, of irregularly spaced data points (containing information about the measured values of the modelled phenomenon), to create a continuous surface of values of the studied values. Modelled area will then be covered with a regular network, determined by the distance between the nodes, with the numbers in each direction and rotation. Selected interpolation methods be gradually filling in the "space" between the positions of data points and unmapped value in nodes positions network [1]. By using interpolation techniques are calculated spatial changes in the measured data in the study area.

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The second data type used for representation of the surface will be triangulated irregular network (TIN). TIN divides the surface into a set of contiguous and non-overlapping triangles and can be constructed of any combination of points, lines or polygons features as well as from grids.

2.1 IDW method

It is a deterministic method, use to calculate the linear weighted average. [7] Weight, which is used in calculating, is the reciprocal of the distance measurement from a local estimate with square of *p*.

Equation of method has the following form [2]:

$$F(x, y) = \sum_{i=1}^{n} w_i f_i \qquad (1)$$

where n is number of measured points, f_i is value of variable in the measured point and w_i is weight.

The general equation for the calculation of weights has the form:

$$w_i = \frac{h_i^{-p}}{\sum_{j=1}^{n} h_j^{-p}} \qquad (2)$$

where p is square coefficient of weight and h_i is distance between measured and interpolated points and is calculated as:

$$h_i = \sqrt{(x - x_i)^2 + (y - y_i)^2} \qquad (3)$$

Where x, y are coordinates of estimated points and x_i and y_i are coordinates of measured points. For the weight coefficient p is often used value 2 (then goes on inverse distance squared), generally are recommended value between 1 and 3. In the primary form it comes to exact interpolation method. It preserves the minimum and maximum of the input dataset.

The interpolated value is obtained from the equation:

$$z_k = \frac{\sum_{i=1}^{n} \frac{f_i}{h_i^p}}{\sum_{i=1}^{n} \frac{1}{h_i^p}} \qquad (4)$$

For this method, generally, the closer points have greater weight than points further away from the interpolated place. In input data creating clusters can occur to the formation of disruptive “bull eyes” effect. By Horák [3] is for relieving this effect is introduced into the formula so called an extermination factor (uncertainty factor as this factor is introduced in the calculation, from exact interpolation method is becoming approximation methods [3].

2.2 Spline

Spline method estimates the unknown values by using mathematical functions. This method is based on two conditions:

- estimated mathematical function imitates the thin flexible surface which is compelled to pass through exactly measuring points (exact),
- this surface must have a minimum curvature.

Thereby More entry points, then the greater the impact on the interpolated value will have remote points and then the surface will be smoother. Used types of this method:

- Regularized spline algorithm creates smoother surface, which gradually changes to the values that lie outside the known entry points.
- Spline algorithm with tension creates a surface that is less smoothed, the values of the entry points are more affected small areas in their near.

2.3 Kriging

It is geostatistical method of estimation [3]. The method does not retain the original value, but made their extermination. Spatial autocorrelation phenomenon with respect to the distance and direction of the action expresses semivariogram. Semi variation in the context of data set $\gamma^*(h)$ is calculated as:

$$\gamma^*(h) = \left[\frac{1}{2n(h)}\right] \sum_{i=1}^{n(h)} \left[Z(x_i + h) - Z(x_i)\right]^2 \qquad (5)$$

Where $Z(x_i)$ value of regionalised variable at a certain point, and n is is number of sample pairs of a reciprocal distance *h*.

Estimated value of unknown variables can be written as [8]:

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$$f(z)=\sum_{i=1}^{n}\lambda_i f(x_i) \qquad (6)$$

where $f(x_i)$ is measured value in i-th point, λ_i is unknown weight measured value in i-th point, z is estimated value, n is number of measured values.

2.4 Triangular Irregular Networks - TIN

To representation of continuous phenomena very often is used triangular irregular network (TIN). It uses the Delaunay triangulation - criterion is called. Delaunay condition, i.e. the inside of the circle is described by an arbitrary triangle does not contain any additional (fourth) point (vertex of another triangle). Each triangle (facet) is defined by three points, which stores information on altitude of the specific location on the X axis and Y. Triangles are irregular and don´t overlap each other.

2.5 ABOS

This method provides an innovative method for the approximation (interpolation) irregularly spaced points in 3D Euclidean space of continuous functions of two independent variables that can be used to create models in many fields of Geosciences, such as geography, geology, meteorology. The author of this method is M. Dressler, which subsequently named ABOS (Approximation / interpolation Based On Smoothing) and uses simple mathematical tools - numeric tension and extermination.

Interpolation function is intended by the matrix P real numbers, whose elements (z-location) are assigned to nodes of regular grid covering the domain D[4].

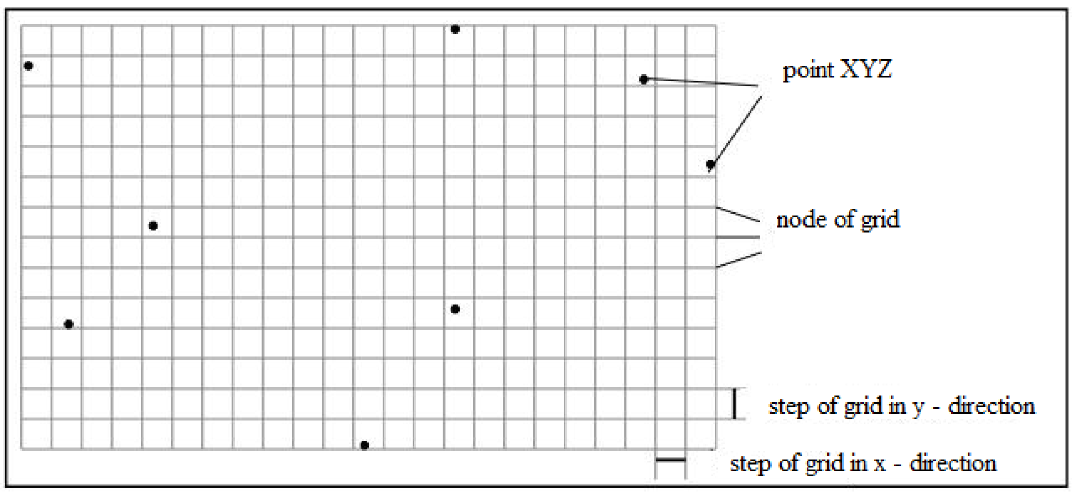

Fig. 1 Regular rectangular grid to define interpolation functions [4]

Value at each point of the surface (x_0, y_0) in the grid can be evaluated from the equation bilinear polynomial,

$$f(x, y) = axy + bx + cy + d, \qquad (7)$$

which is defined by the corner points of the grid point (x_0, y_0).

III. INPUT DATA AND SOFTWARE

Input data for processing and analysis are the spatial coordinates X, Y, Z 266 points measured at the heap (see Figure.2). These points constitute boundary of heap curves and were focused in the area around Košice (GNSS measurement).

Feb , 2014. Vol. 4, No. 8 ISSN2305-8269

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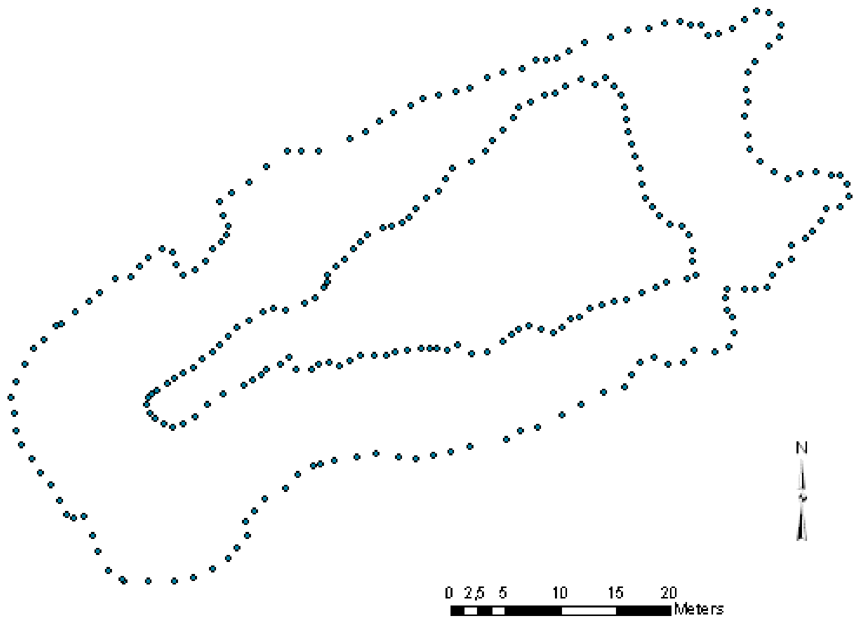

Fig. 2 Measured detailed points

At present there is much software that enable us to interpolate geodata. By Application Spatial Analyst were used implemented interpolation methods. For IDW method was used square coefficient of weight 2, i.e. it is a method of IDS. To estimate the values were used other methods:

- method of regularized spline,
- spline with tension method,
- kriging estimates were made on the basis of triangulated irregular network TIN.

Author of the chosen method is also the author of a program that contains a number of features that can be generated surface and subsequently convert various spatial analyses.

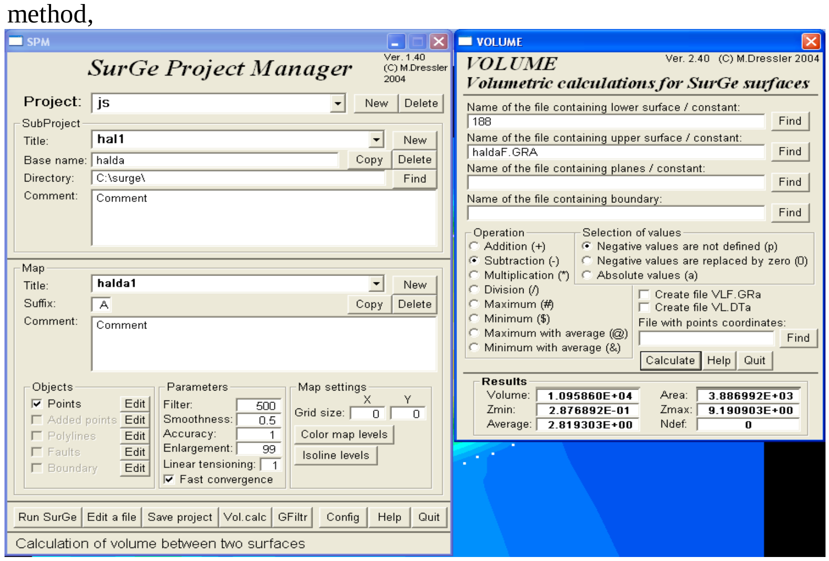

Fig. 3 Software Environment of Programme SURGE [6]

Feb , 2014. Vol. 4, No. 8 ISSN2305-8269

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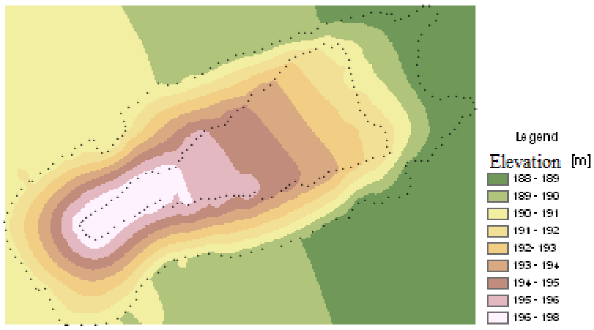
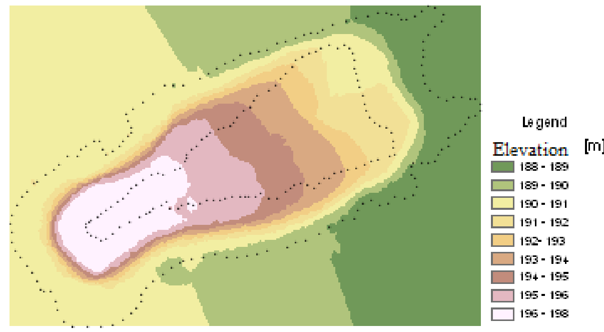
Modelling was performed by using predefined settings grid size 500×338. For smoothing parameter was set to a value of 0.5, i.e. value for normal interpolation.

IV. RESULTS

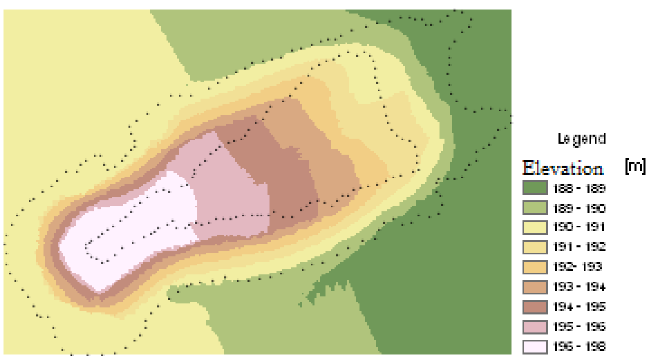
By using these methods we got the resulting surface of examined heap (see Figure. 4).



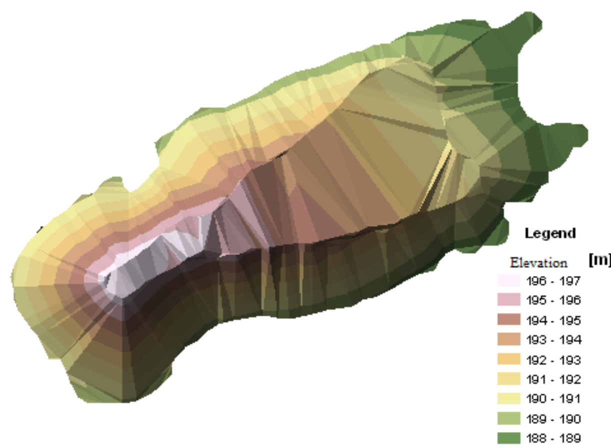
Interpolation by ABOS method



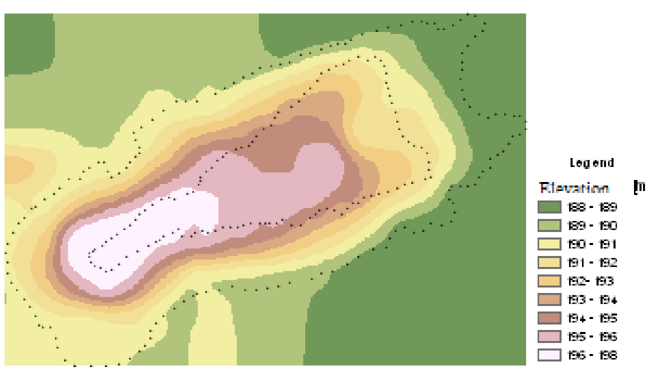
IDS interpolation



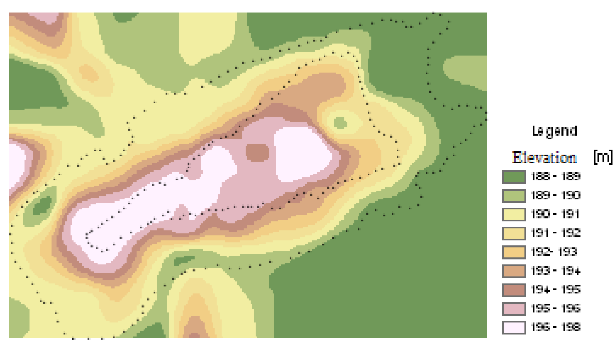
Kriging



TIN



Spline with tension



Regularized spline

Fig. 4 Visualization of interpolation results

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From the presented results, it is seen, that the method ABOS can be display surface sufficient with reality. Used program SurGe enables us to render a wide variety of surface shapes. Very simply enables render isolines (see Figure. 5) and also create a 3D model of the terrain surface (see Figure. 6).

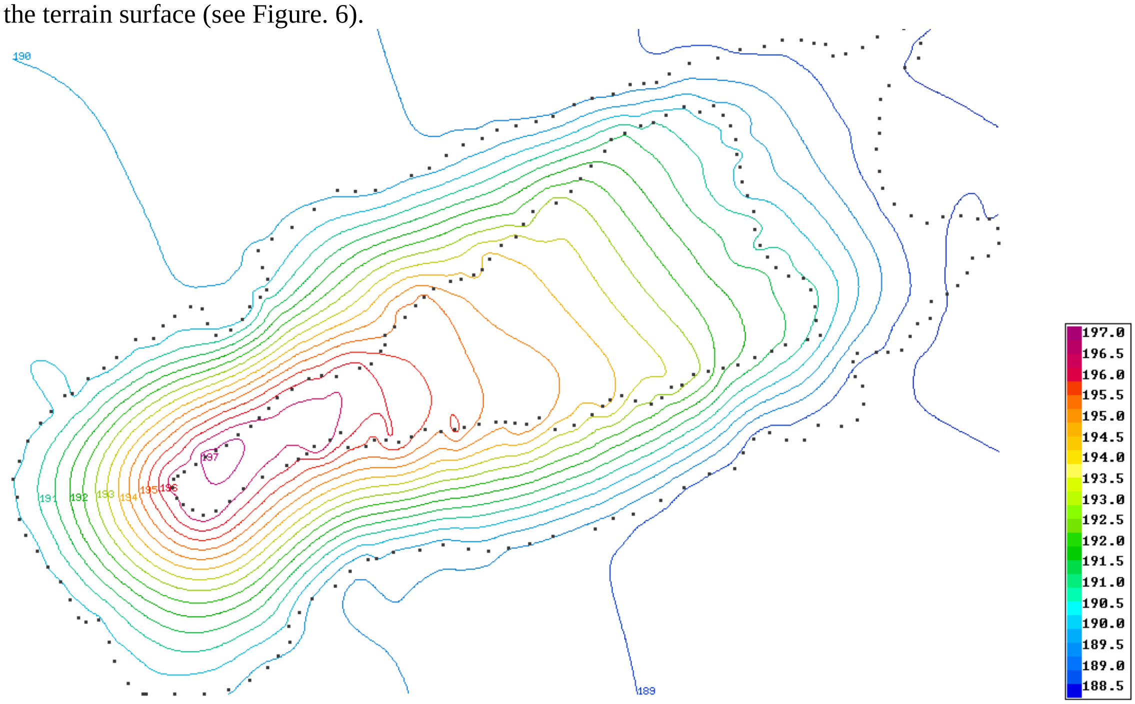

Fig. 5 Isolines of measured heap

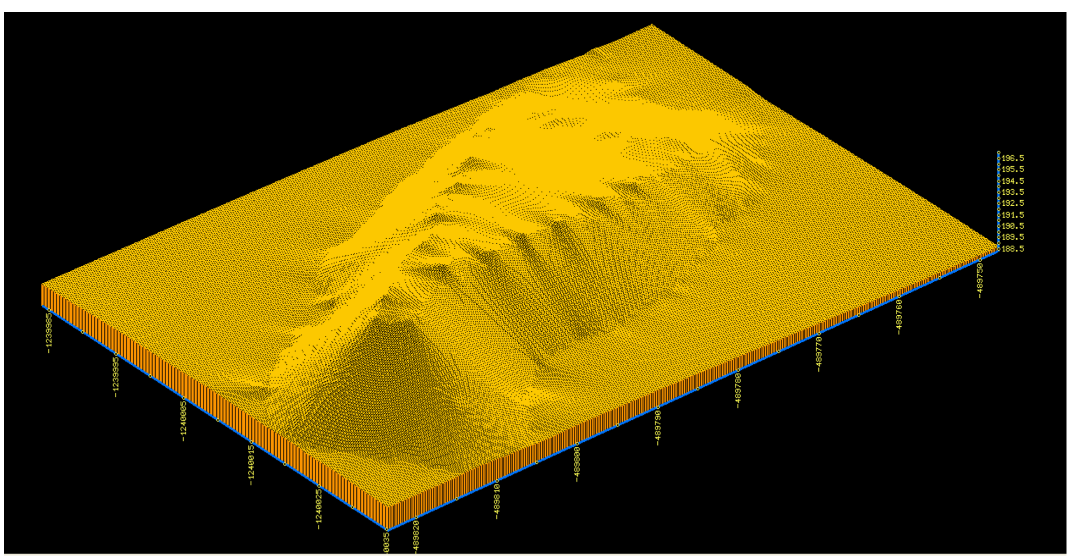

Fig. 6 3D view of the measuring sample

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For each method were calculated following statistical indicators (see Table. 1).

Tab. 1 Assessment of the accuracy of each method

Method	*The average absolute error [m]*	*The mean square error RMS [m]*
IDW	0,00000023	0,00642437
Kriging	0,00005140	0,17193661
Spline with tension	0,00000378	0,00359425
Spline reg.	0,00000126	0,001964808
ABOS	0,00001099	0,01171924
TIN	0,00004381	0,02409946

Generated surfaces heap were evaluated among themselves by using a profile rendered in the selected line. The result is the graph of each method (see Figure. 7).

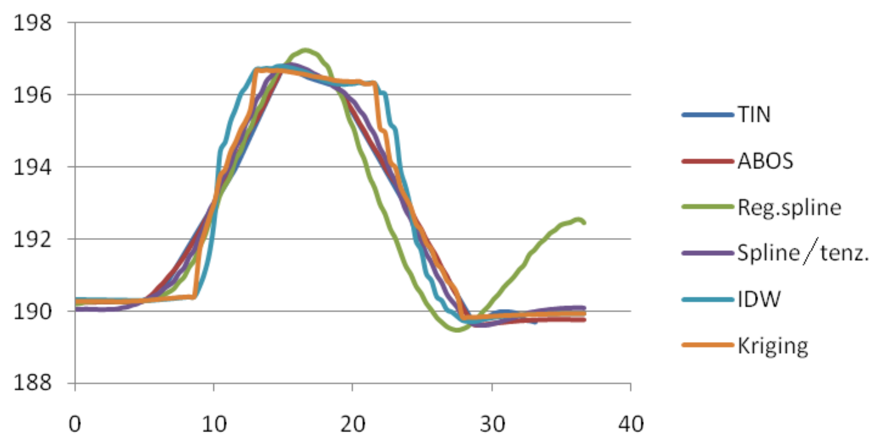

Fig. 7 Profiles comparison of individual method

Displayed heap surface (see Figure. 6) was used to calculate the volume. Value of calculation of the heap calculated by using various interpolation methods indicated in Table 2.

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Tab. 1 Volume of heap calculated through selected interpolation methods

Method	Volume [m^3]
ABOS	7485
IDW	7522
Reg.spline	7499
Spline with tension	7607
Kriging	7507
TIN	7598

V. CONCLUSION

Analyses of models represent the mathematical formulation of various processes. It should be realized that the presented results are always dependent on the document and interpolation methods on which the model is made [5]. In addition to the interpolation method has a significant impact on the accuracy of modelling the origin of the input data. The presented results show the diversity of the interpolated surface, which emphasizes the importance of choosing the interpolation method. Diversity of algorithm of these methods gives us the different results in the calculations the volume of the sample examined, as demonstrated values specified in Table. 2 of contribution.

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