April, 2014. Vol. 4, No. 11 ISSN2305-8269
International Journal of Engineering and Applied Sciences

www.eaas-journal.org

SHORTEST DISTANCE FROM A POINT TO TRIAXIAL ELLİPSOİD

Sebahattin Bektaş[1]

Abstract

Finding the shortest distance to a triaxial ellipsoid is equivalent to the presence of ellipsoidal heights. The problem of finding the shortest distance problems are encountered frequently in the Cartesian- Geodetic coordinate transformation,optimization problem , fitting ellipsoid ,image processing, face recognition, computer games etc. We have chosen as a triaxial ellipsoid surface of reason is that a general surface. Thus, the minimum distance from biaxial ellipsoid and sphere is found with same algorithm . In this paper we study on the computation the shortest distance from a point to a triaxial ellipsoid.

Key words: *Shortest distance, Triaxial ellipsoid, Cartesian- Geodetic coordinate, Coordinate transformation,System of nonlinear equations*

1.Introduction

Although Triaxial ellipsoid equation is quite simple and smooth but geodetic computations is quite difficult on the Triaxial ellipsoid. The main reason for this difficulty is the lack of symmetry. Triaxial ellipsoid generally not used in geodetic applications. Rotational ellipsoid (ellipsoid revolution ,biaxial ellipsoid , spheroid) frequently used in geodetic applications .

First, the basic definitions triaxial ellipsoid start giving mathematical equations to explaining the concepts. To show how computations the shortest distance to Triaxial ellipsoid. The efficacy of the new algorithms is demonstrated through simulations.

Triaxial ellipsoid formula is more useful formulas. Because of to obtain the rotational ellipsoid formula from Triaxial ellipsoid formula is

$$\frac{x^2}{a_x^2}+\frac{y^2}{a_y^2}+\frac{z^2}{b^2}=1 \qquad (1)$$

The following definitions will be used.

a_x = equatorial semimajor axis of the ellipse
a_y = equatorial semiminor axis of the ellipse
b = polar semi-minor axis of the ellipse
λ = geodetic longitude
φ = geodetic latitude
h = ellipsoid height : the shortest distance

Finding The Shortest Point On The Ellipsoid

easily. For this equatorial semi-axis of triaxial ellipsoid formula a_x=a_y =a is sufficient. Similarly to obtain sphere of formula from ellipsoid formula a = b= R is sufficient Bektas(2009).

Ellipsoid

An ellipsoid is a closed quadric surface that is analogue of an ellipse (see Fig.1). Ellipsoid has three different axes (a_x>a_y>b). Mathematical literature often uses “ellipsoid” in place of “Triaxial ellipsoid or general ellipsoid”. Scientific literatura (particularly geodesy) often uses “ellipsoid” in place of “biaxial ellipsoid,rotational ellipsoid or ellipsoid revolution”. Older literature uses ‘spheroid’ in place of rotational ellipsoid. The standart equation of an ellipsoid centered at the origin of a cartesian coordinate system and aligned with the axes Fig.1.

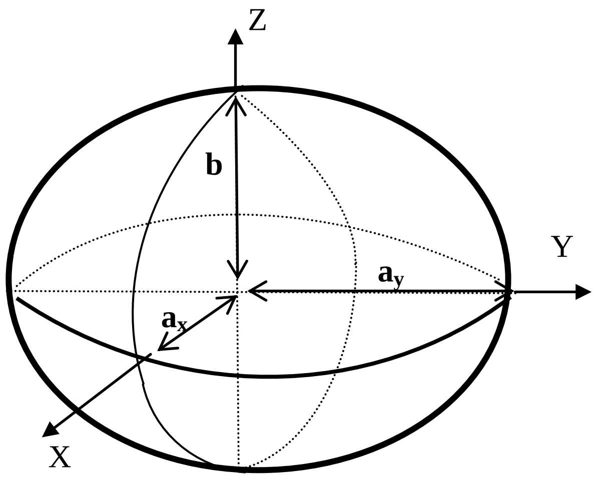

Figure-1 Ellipsoid

April, 2014. Vol. 4, No. 11 ISSN2305-8269
International Journal of Engineering and Applied Sciences

www.eaas-journal.org

It can be proved that the shortest distance is along the surface normal. The first step is to find the projection of an external point denoted as $P_G(x_G, y_G, z_G)$ in Fig.2 onto this ellipsoid along the normal to this surface i.e. point P_E (x_E, y_E, z_E) Feltens ,J. 2009, (J Geod 83:129-137) , Ligas,M. 2012 ,(J Geod 86:249–256)

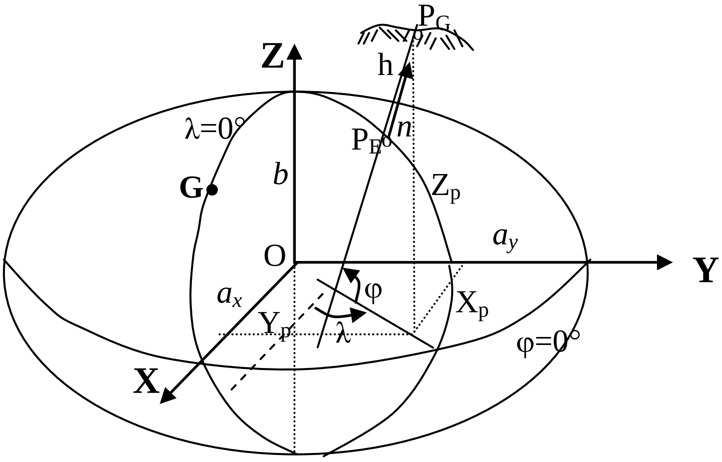

Figure-2 (X,Y,Z) Cartesian (x,y,z) and Geodetic (φ, λ, h) coordinates on Triaxial ellipsoid

Feltens (2009) gives a vector-based iteration process for finding the point on the ellipsoid. Ligas(2012) claims his methods turn out to be more accurate, faster and applicable. Licas Methods is based on solving nonlinear system of equation. He established three nonlinear sytem of equation (Case 1, Case 2, Case 3),there are 3 equation in each case. He solve one of them using Newton's methods. And then the results which he obtained from the solution of equations were iteratived. But it was not clear which system were selected in that case.

The presented method is based on solving nonlinear system of equation too. But the system is univocal and has got 4 equation with 3 unknows, and also the system is overdetemined

Accordingly to Fig.4 can be writing a colinearite condition between P_E and P_G

$$\mathbf{n} = [n_1, n_2, n_3] = grad\phi = \left[\frac{\partial\phi}{\partial x_E}, \frac{\partial\phi}{\partial y_E}, \frac{\partial\phi}{\partial z_E}\right] = 2\left[\frac{x_E}{a_x^2}, \frac{y_E}{a_y^2}, \frac{z_E}{b^2}\right] = 2[E.\,x_E\,, F.y_E\,, G.z_E\,]$$

The first stage begins with constructing two collinear vectors: a vector normal (n) to the ellipsoid (obtained from the gradient operator of a triaxial ellipsoid) in the point P_E what may be expressed as (seen in Fig. 2):

$$E = \frac{1}{a_x^2}; \qquad F = \frac{1}{a_y^2}; \qquad G = \frac{1}{b^2}$$

$$\phi(x_E, y_E, z_E) = \frac{x_E^2}{a_x^2} + \frac{y_E^2}{a_y^2} + \frac{z_E^2}{b^2} - 1 = 0 \qquad (2)$$

and a vector (h) the shortest distance, connecting points P_G and P_E, Fig. 2:

$$h = [h_1, h_2, h_3] = [x_E - x_G, y_E - y_G, z_E - z_G] \quad (3)$$

From the essentials of vector calculus it is known, that coordinates of collinear vectors are proportional with some constant factor k, thus, we may write:

April, 2014. Vol. 4, No. 11 ISSN2305-8269
International Journal of Engineering and Applied Sciences

www.eaas-journal.org

$$k = \frac{h_1}{n_1} = \frac{h_2}{n_2} = \frac{h_3}{n_3} \Rightarrow k = \frac{x_E - x_G}{E.x_E} = \frac{y_E - y_G}{F.y_E} = \frac{z_E - z_G}{G.z_E} \qquad (4)$$

From the above three equations are obtained, i.e.:

$f1 = (x_E - x_G) F. y_E -(y_E - y_G) E. x_E$ (5.a)
$f2 = (x_E - x_G) G. z_E -(z_E - z_G) E. x_E$ (6.b)
$f3 = (y_E - y_G) G. z_E -(z_E - z_G) F. y_E$ (6.c)
$f4 = E.x_E^2 + F.y_E^2 + G.z_E^2 -1$ (6.d)

If these four equations linearized by Taylor series expansion, the system of equations to be solved in order to obtain the solution for $X_E = [x_E , y_E , z_E]$.

The initial guesses for the point on the ellipsoid P_E were chosen the same as in Feltens (2009), namely:

$$x_E^o = \frac{a_x.x_G}{\sqrt{x_G^2 + y_G^2 + z_G^2}} \qquad y_E^o = \frac{a_y.y_G}{\sqrt{x_G^2 + y_G^2 + z_G^2}} \qquad z_E^o = \frac{b.z_G}{\sqrt{x_G^2 + y_G^2 + z_G^2}} \qquad (7)$$

The entries may be written as follows

$(x_o = x_E^o \qquad y_o = y_E^o \qquad z_o = z_E^o)$

j_{11}=F.yo-(yo-y).E; j_{12}=(xo-x).F-E.xo; j_{13}=0;

j_{21}=G.zo-(zo-z).E; j_{22}=0; j_{23}=(xo-x).G-E.xo;

j_{31}=0; j_{32}=G.zo-(zo-z).F; j_{33}=(yo-y).G-F.yo;

j_{41}=2.E.xo j_{42}=2F.yo j_{43}=2G.zo

f_1 = (xo-x).F.yo-(yo-y).E.xo (8.a)
f_2 = (xo-x).G.zo-(zo-z).E.xo (8.b)
f_3 = (yo-y).G.zo-(zo-z).F.yo (8.c)
f_4 = $E.xo^2 + F.yo^2 + G.zo^2 - 1$ (8.d)

$$A = \begin{bmatrix} j_{11} & j_{12} & j_{13} \\ j_{21} & j_{22} & j_{23} \\ j_{31} & j_{32} & j_{33} \\ j_{41} & j_{42} & j_{43} \end{bmatrix} \qquad \delta_E = \begin{bmatrix} \delta x_E \\ \delta y_Y \\ \delta z_Z \end{bmatrix} \qquad f = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} \qquad (9)$$

$A\, \delta_i + f = 0$ (10)

can be solved very easily in MATLAB

$\delta_i = - A \backslash f$ (11)

or clasically

$\delta_i = - (A^T A)^{-1}.A^T f$ (12)

Thus an iterative solution scheme may be implemented by:

$\mathbf{X}_{\mathbf{E}i+1} = \mathbf{X}_{\mathbf{E}i} + \delta_i$ (13)

International Journal of Engineering and Applied Sciences

www.eaas-journal.org

If δ_i is less than threshold stop the iteration. Our presented solved method is a kind of least squares adjustement. This is based on combined least squares method. After the first step is accomplished, founding P_E on the ellipsoid, the solution h found directly below formulas:
Also, having coordinates of P_E the shortest distance P_G P_E,= h may easily be computed as:

$$h = sign(z_E - z_G).sign(z_E)\sqrt{(x_E - x_G)^2 + (y_E - y_G)^2 (z_E - z_G)^2}$$

In order to demonstrate the validty of the shortest distance algorithms presented above ,a numerical example are given . The algorithm were implemented in MATLAB. The numerical computations in the triaxial case were carried out using Earth's geometrical parameters
a_x= 6378388.0000 a_y = 6378318.0000 b= 6356911.9461

Numeric Example:
Given P_G point Cartesian coordinates

x_G = 3909863.9271 , y_G =3909778.1230 and z_G = 3170932.5016

Finding the shortest distance to triaxial ellipsoid.
Fort this firstly must find P_E point cartesian coordinates. The initial guesses for the P_E point on the ellipsoid Eq.(7)

$$x_E^o = \frac{a_x.x_G}{\sqrt{x_G^2 + y_G^2 + z_G^2}} = 3909255.6655$$

$$y_E^o = \frac{a_y.y_G}{\sqrt{x_G^2 + y_G^2 + z_G^2}} = 3909169.8616$$

$$z_E^o = \frac{b.z_G}{\sqrt{x_G^2 + y_G^2 + z_G^2}} = 3170435.9034$$

finding P_E point on the ellipsoid from P_G point iteratevly from Eqs(10-11)

i	Xe_i	Ye_i	Ze_i	δx_i	δy_i	δz_i
1	3909255.6655	3909169.8616	3170435.9034	-3283.630	-3240.634	8017.501
2	3909251.5547	3909165.7506	3170432.5016	-4.111	-4.111	-3.402
3	3909251.5547	3909165.7506	3170432.5016	-0.000	-0.000	-0.000

x_E = 3909251.5547 y_E =3909165.7506 z_E =3170432.5016

$$h = sign(z_E - z_G).sign(z_E)\sqrt{(x_E - x_G)^2 + (y_E - y_G)^2 (z_E - z_G)^2}$$

h =1000.000m.

5.Conclusion

In this paper we study on the computation the shortest distance from a point to a triaxial ellipsoid. The problem of finding the shortest distance problems are encountered frequently in the Cartesian- Geodetic coordinate transformation,optimization problem , fitting ellipsoid ,image processing, face recognition, computer games etc. The paper has presented a new method of shortest distance to a triaxial ellipsoid. The new method relies on solving a overdetermined system of nonlinear equations with the use of a generalized Newton method. It has been compared to the other existing methods. In conclusion,the presented method may be considered as fast , accurate and reliable and may be successfully used in other areas.The presented algorithm can be applied easily for biaxial ellipsoid,and sphere,also other surface such as paraboloid,hyperboloid

ISSN2305-8269
International Journal of Engineering and Applied Sciences

www.eaas-journal.org

6.References

1. Bektas,S. (2009). "Practical Geodesy", Ondokuz Mayis University Press,ISBN 978-975-7636-65-6,Samsun

2. Feltens J (2009) Vector method to compute the Cartesian (*X*, *Y* , *Z*) to geodetic (φ, *λ*, *h*) transformation on a triaxial ellipsoid. J Geod 83:129–137

3. Ligas M., (2012a) Cartesian to geodetic coordinates conversion on a triaxial ellipsoid, J. Geod., 86, 249-256.

4. Ligas M., (2012b), Two modified algorithms to transform Cartesian to geodetic coordinates on a triaxial ellipsoid, Stud.Geoph. Geod., 56, 993-1006.

5. Weisstein, Eric W. "Ellipsoid." From *MathWorld*--A Wolfram Web Resource. http://mathworld.wolfram.com/Ellipsoid.html