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A NEW SOLUTION OF SIR MODEL BY USING THE DIFFERENTIAL FRACTIONAL TRANSFORMATION METHOD

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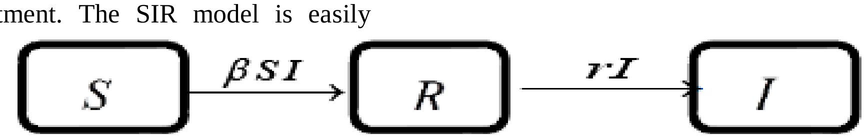
ABSTRACT

In this paper, we introduce a new solution of the SIR model by using the differential fractional transformation method. We study the difference between the integral Riemann-Liouville, differential Riemann-Liouville and the Caputo fractional derivative. We use some theorems of the fraction to introduce the solution of SIR model. Numerical results have provided to confirm the theoretical result and the efficiency of the proposed method.

Keywords: Caputo and Riemann-Liouville of Fractional; Theorems of Fractional; SIRModel; DFTM

1. INTRODUCTION

A major assumption of many mathematical models of epidemics is that the population can be divided into a set of distinct compartments. These compartments are defined with respect to disease status. The simplest model, which was described by Kermack and McKendrick in1927a [16-18], consists of three compartments: susceptible (S), infected (I), recovered (R). Susceptible Individuals that are susceptible have, in the case of the basic SIR model, never been infected, and they are able to catch the disease. Once they have it, they move into the infected compartment. Infected individuals can spread the disease to susceptible individuals. The time they spend in the infected compartment is the infectious period, after which they enter the recovered compartment. The SIR model is easily written using ordinary differential equations (ODEs), which implies deterministic model (no randomness is involved, the same starting conditions give the same output), with continuous time (as opposed to discrete time). Analogous to the principles of reaction kinetics, we assume that encounters between infected and susceptible individuals occur at a rate proportional to their respective numbers in the population. The rate of new infections can thus be defined as βSI , where β is a parameter for infectivity. Infected individuals are assumed to recover with a constant probability at any time, which translates into a constant per capita recovery rate that we denote with r , and thus an overall rate of recovery rI . Based on these assumptions we can draw the scheme of the model.



The scheme can also be translated into a set of differential equations [13, 14]:

$$\frac{dS}{dt} = -\beta SI$$

$$\frac{dI}{dt} = \beta SI - rI$$

$$\frac{dR}{dt} = rI. \qquad (1.1)$$

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Using this model, we will consider a mild, short-lived epidemic, e.g. influenza, in a closed population. Closed means that there is no immigration or emigration. Moreover, given the time scale of influenza epidemics, we will not consider demographic turnover (birth or death), and all infections are assumed to end with recovery. The size of the population ($S+I+R$) is therefore constant and equal to the initial population size, which we denote with the parameter N .

Let us now consider a population which is naive with respect to the disease we are consider. What happens if a single infected individual is introduced into such a population? Is there going to be an epidemic? How many people will be infected? We will answer these questions by implementing and simulating the model in R .

The differential transformation method is a numerical method based on a Taylor expansion. This method constructs an analytical solution in the form of a polynomial. Differential Transform Method (DTM) is one of the analytical methods for differential equations. The basic idea was initially introduced by Zhou [9] in 1986. Its main application therein is to solve both linear and nonlinear initial value problems in electrical circuit analysis.

This method develops a solution in the form of a polynomial. Though it is based on Taylor series, still it is totally different from the traditional higher order Taylor series method. The DTM is an alternative procedure for getting Taylor series solution of the differential equations. This method reduces the size of computational domain and is easily applicable to many problems. Large list of methods, exact, approximate and purely numerical are available for the solution of differential equations. Most of these methods are computationally intensive because they are trial-and error in nature, or need complicated symbolic computations. The differential transformation technique is one of the numerical methods for ordinary differential equations. This method constructs a semi-analytical numerical technique that uses Taylor series for the solution of differential equations in the form of a polynomial. It is different from the high-order. Taylor series method which requires symbolic computation of the necessary derivatives of the data functions. The Taylor series method is computationally time-consuming especially for high order equations. The differential transform is an iterative procedure for obtaining analytic Taylor series solutions of differential equations. The main advantage of this method is that it can be applied directly to nonlinear ODEs without requiring linearization, perturbation. This method will not consume too much computer time when applying to nonlinear or parameter varying systems. This method gives an analytical solution in the form of a polynomial. But, it is different from Taylor series method that requires computation of the high order derivatives. The differential transform method is an iterative procedure that is described by the transformed equations of original functions for solution of differential equations. In recent decades, many effective methods have been established for solutions of differential equations , such as the two-boundary-value problems [1], the two-dimensional differential transform method [2], the optimization of the rectangular fins [3-5], the initial value problems [6-8], the Adomain decomposition method [10], and so on. Unlike the traditional high order Taylor series method which requires a lot of symbolic computations, the differential transform method is an iterative procedure for obtaining Taylor series solutions.

The rest of this article is organized as follows: In Sec.2, basic definitions and theorems of fractional are given. In Sec.3, we apply the solution of SIR model. In Sec.4, numerical results are given. In Sec.5, some conclusions are given.

2. BASIC DEFINITIONS AND THEOREMS OF FRACTIONAL

There are several definitions of a fractional derivative of order α >0 [11, 12],e. g. Riemann-Liouville, Grunwald-Letnikow, Caputo and generalized functions approach. The most commonly used definitions are the Riemann-Liouville and Caputo. We give some basic definitions and properties of the fractional calculus theory which are used further in this paper.

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Definition 1: The Riemann-Liouville fractional derivative operator D_R^{α} of order α is defined by

$$D_R^{\alpha} f(x) = \begin{cases} \dfrac{1}{\Gamma(m-\alpha)} \dfrac{d^m}{dx^m} \displaystyle\int_0^x \frac{f^m(t)}{(x-t)^{\alpha-m+1}} dt & m-1<\alpha<m \\ \dfrac{d^m f(x)}{dx^m} & \alpha = m \end{cases} . \quad (2.1)$$

Definition 2: The Caputo fraction derivative operator D_R^{α} of order α is defined in the following

Form [12]:

$$D^{\alpha} f(x) = \frac{1}{\Gamma(m-\alpha)} \int_0^x \frac{f^m(t)}{(x-t)^{\alpha-m+1}} dt, \qquad \alpha > 0, \quad (2.2)$$

where $m-1<\alpha<m, for \ m \in N, \ x>0.$

Similar to integer order differentiation, Caputo fractional derivative operator is a linear operation

$$D^{\alpha}(\lambda f(x) + \mu g(x)) = \lambda D^{\alpha} f(x) + \mu D^{\alpha} g(x), \quad (2.3)$$

where λ, μ are constant.

Also Caputo fractional derivative can affect on constant is constant $D^{\alpha} C = 0$, C
For the Caputo's derivative we have

$$D^{\alpha} x^n = \begin{cases} 0 & for \ n \in N_0 \ and \ n < \lceil \alpha \rceil \\ \dfrac{\Gamma(n+1)}{\Gamma(n+1-\alpha)} x^{n-\alpha} & for \ n \in N_0 \ and \ n \geq \lceil \alpha \rceil \end{cases} . \quad (2.4)$$

We use the ceiling function $\lceil \alpha \rceil$ to denote the smallest integer than or equal to and α . $N_0 = \{0,1,2,...\}$ Recall that for $\alpha \in N$, the Caputo differential operator coincides with usual differential operator of integer order. The following theorem that are given below, for proofs and details see ([21], [22])

Theorems 1: If $f(x) = g(x) \pm h(x)$ then, $F(K) = G(K) \pm H(K).$

Theorem 2: If $f(x) = g(x)h(x)$ then, $F(K) = \sum_{l}^{K} G(l) H(K-l).$

Theorem 3: If $f(x) = g_1(x) g_2(x) \ldots\ldots g_{n-1}(x) g_n(x)$ then

$$F(K) = \sum_{K_{n-1}=0}^{K} \sum_{K_{n-2}=0}^{K_{n-1}} \ldots\ldots \sum_{K_2=0}^{K_3} \sum_{K_1=0}^{K_2} G_1(K_1) G_2(K_2-K_1) \ldots\ldots G_{n-1}(K_{n-1}-K_{n-2}) G_n(K_{n-1}-K_n).$$

Theorem 4: If $F(K) = \delta(K-\alpha p)$ then, $f(x) = (x-x_0)^p$

$$\delta(K) = \begin{cases} 1 & if \ K=0 \\ 0 & if \ K \neq 0 \end{cases} .$$

Theorem 5: I $f(x) = D_{x_0}^q [g(x)]$ then, $F(K) = \dfrac{\Gamma(q+1+K/\alpha)}{\Gamma(1+K/\alpha)} G(K+\alpha q).$

Now let us expand the analytic and continuous function $f(x)$ in terms of fractional power series as follows:

$$f(x) = \sum_{K=0}^{\infty} F(K)(x-x_0)^{K/\alpha}. \tag{2.5}$$

To transform the initial condition to functions, we use the relation:-

$$F(K) = \begin{cases} if \;\; K/\alpha \in Z^+, & \dfrac{1}{(K/\alpha)!}\left[\dfrac{d^{K/\alpha} f(x)}{dx^{K/\alpha}}\right], \;\; K = 0,1,2......(\alpha q - 1), \\ f \;\; K/\alpha \notin Z^+, & 0. \end{cases} \tag{2.6}$$

where q is the order of fractional differential considered.

3. SOLUTION OF SIR MODEL

We now introduce fractional order into the model. The new system is described by the following set of FDE of order $\alpha_1, \alpha_2, \alpha_3 \succ 0$.

$$\begin{aligned} D^{\alpha_1}(S) &= (1-\mathrm{p})\pi - \beta SI - \pi S \\ D^{\alpha_2}(I) &= \beta SI - (\gamma + \pi) I \\ D^{\alpha_3}(R) &= \mathrm{p}\,\pi + \gamma I - \pi R, \end{aligned} \tag{3.1}$$

with the initial conditions $S(0) = 0, \;\; I(0) = 1, \;\; R(0) = 1.$ (3.2)

By using theorems (1), (2) and (5) we get

$$\begin{aligned} S(K+\alpha_1\beta_1) &= \frac{\Gamma(1+K/\beta_1)}{\Gamma(\alpha_1+1+K/\beta_1)}\left\{(1-P)\pi - \left[\beta \sum_{l=0}^{K} S(l)I(K-l)\right] - \pi S(K)\right\} \\ \mathrm{I}(K+\alpha_2\beta_2) &= \frac{\Gamma(1+K/\beta_2)}{\Gamma(\alpha_2+1+K/\beta_2)}\left\{\beta \sum_{l=0}^{K} S(l)I(K-l) - (\gamma+\pi)I(K)\right\} \\ \mathrm{R}(K+\alpha_3\beta_3) &= \frac{\Gamma(1+K/\beta_3)}{\Gamma(\alpha_3+1+K/\beta_3)}\left\{P\pi + \gamma I(K) - \pi R(K)\right\} \end{aligned} \tag{3.4}$$

$\beta_1, \beta_2, \beta_3$ are the unknown values of the fractions of $\alpha_1, \alpha_2, \alpha_3$ respectively also by using (3.2) and (3.4) we have got

$$\begin{aligned} &S(K) = 0 && for \;\; K = 0,1,2,.....,\alpha_1\beta_1 - 1 \\ &I(K) = 0 && for \;\; K = 0,1,2,.....,\alpha_2\beta_2 - 1 \\ &R(K) = 0 && for \;\; K = 0,1,2,.....,\alpha_3\beta_3 - 1 \\ &I(0) = 1, && R(0) = 1. \end{aligned} \tag{3.5}$$

Using the latest transform equation $S(K)$ for $K = \alpha_1\beta_1, \alpha_1\beta_1 - 1,...., n$, $I(K)$ for $K = \alpha_2\beta_2, \alpha_2\beta_2 - 1, ..., \mathrm{n}$

and $R(K)$ for $K = \alpha_3\beta_3, \alpha_3\beta_3 - 1, ..., \mathrm{n}$.

are calculated and using the inverse transformation rule at $\alpha_1 = \alpha_2 = \alpha_3 = 1$.

Put $K=0$

$$S(0+1) = S(1) = \frac{\Gamma(1+0)}{\Gamma(1+1+0)}\{(1-P)\pi - \beta S(0)I(0) - \pi S(0)\}$$

$$= \{(1-P)\pi - \beta.0.1 - \pi.0\} = (1-P)\pi$$

$$\mathrm{I}(0+1) = I(1) = \frac{\Gamma(1+0)}{\Gamma(1+1+0)}\{\beta S(0)I(0) - (\gamma+\pi)\mathrm{I}(0)\}$$

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$$=-(\gamma+\pi)$$

$$R(0+1)=R(1)=\frac{\Gamma(1+0)}{\Gamma(1+1+0)}\{P\pi+\gamma I(0)-\pi R(0)\}$$

$$=P\pi-\pi+\gamma=\pi(P-1)+\gamma.$$

Put K=1

$$S(1+1)=S(2)=\frac{\Gamma(1+1)}{\Gamma(1+1+1)}\{(1-P)\pi-\beta S(1)I(0)-\pi S(1)\}$$

$$=\frac{1}{2}\left\{(1-p)\pi-\beta\pi(1-p)-\pi^2(1-p)\right\}=\frac{\pi}{2}\{(1-p)[1-\beta-\pi]\}$$

$$I(1+1)=I(2)=\frac{\Gamma(1+1)}{\Gamma(1+1+1)}\{\beta S(1)I(0)-(\gamma+\pi)I(1)\}$$

$$=\frac{1}{2}\left\{(\beta\pi(1-p)+(\gamma+\pi)^2\right\}$$

$$R(1+1)=R(2)=\frac{\Gamma(1+1)}{\Gamma(1+1+1)}\{P\pi+\gamma I(1)-\pi R(1)\}$$

$$=\frac{1}{2}\left\{P\pi-\gamma(\gamma+\pi)-\pi\gamma-\pi^2(p-1)\right\}.$$

Put K=2

$$S(2+1)=S(3)=\frac{\Gamma(1+2)}{\Gamma(1+1+2)}\{(1-P)\pi-\beta S(2)I(0)-\pi S(2)\}$$

$$=\frac{1}{3}\left\{(1-P)\pi-\frac{1}{2}\beta[\pi(1-P)(1-\beta-\pi)]-\frac{1}{2}\pi^2(1-P)(1-\beta-\pi)\right\}$$

$$=\frac{\pi}{6}\{(1-P)(2-(1-\beta-\pi)[-\beta-\pi])\}$$

$$I(2+1)=I(3)=\frac{\Gamma(1+2)}{\Gamma(1+1+2)}\{\beta S(2)I(0)-(\gamma+\pi)I(2)\}$$

$$=\frac{1}{3}\left\{\frac{\pi}{2}\beta[(1-P)(1-\beta-\pi)]-\frac{1}{2}(\gamma+\pi)(\beta\pi(1-P)+(\gamma+\pi)^2)\right\}$$

$$=\frac{\pi}{6}\left\{\beta(1-P)(1-\beta-2\pi-\gamma)-(\gamma+\pi)^3\right\}.$$

$$R(2+1)=R(3)=\frac{\Gamma(1+2)}{\Gamma(1+1+2)}\{P\pi+\gamma I(2)-\pi R(2)\}$$

$$=\frac{1}{3}\left\{P\pi+\frac{1}{2}\gamma[\beta\pi(1-P)+(\gamma+\pi)^2-\frac{1}{2}\pi[p\pi-\gamma(\gamma+2\pi)-\pi^2(p-1)]\right\}$$

$$=\frac{\pi}{6}\left\{p(2-\pi)+(1-p)(\gamma\beta+\pi^2)+\gamma[(\gamma+\pi)^2-\gamma-2\pi\right\}.$$

So we have got the solution as the following:

$$S(t)=(1-P)\pi t+\frac{\pi}{2}\{(1-p)[1-\beta-\pi]\}t^2+\frac{\pi}{6}\{(1-P)(2-(1-\beta-\pi)(-\beta-\pi))\}t^3+.....$$

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$$I(t)=1-(\gamma+\pi)\,\mathrm{t}+\frac{1}{2}\left\{(\beta\pi(1-p)+(\gamma+\pi)^2\right\}t^2+\frac{\pi}{6}\left\{\beta(1-P)(1-\beta-2\pi-\gamma)-(\gamma+\pi)^3\right\}t^3+.....,$$

and

$$R(t)=1+(\pi(\mathrm{P}-1)+\gamma)\,\mathrm{t}+\frac{1}{2}\left\{P\pi-\gamma(\gamma+\pi)-\pi\gamma-\pi^2(p-1)\right\}t^2+\frac{\pi}{6}\left\{p(2-\pi)+(1-p)(\gamma\beta+\pi^2)+\gamma[(\gamma+\pi)^2-\gamma-2\pi\right\}t^3+....$$

OBVIOUSLY TABLE 1

K	S(K)	I(K)	R(K)
0	$(1-P)\pi$	$-(\gamma+\pi)$	$(\pi(\mathrm{P}-1)+\gamma)$
1	$\frac{\pi}{2}\{(1-p)[1-\beta-\pi]\}$	$\frac{1}{2}\left\{(\beta\pi(1-p)+(\gamma+\pi)^2\right\}$	$\frac{1}{2}\left\{P\pi-\gamma(\gamma+\pi)-\pi\gamma-\pi^2(p-1)\right\}$
2	$\frac{\pi}{6}\{(1-P)(2-(1-\beta-\pi)(-\beta-\pi))\}$	$\frac{\pi}{6}\left\{\beta(1-P)(1-\beta-2\pi-\gamma)-(\gamma+\pi)^3\right\}$	$\frac{\pi}{6}\left\{p(2-\pi)+(1-p)(\gamma\beta+\pi^2)+\gamma[(\gamma+\pi)^2-\gamma-2\pi\right\}$

4. NUMERICAL RESULTS

We can use the fractional differential to obtain the solution of SIR model for fractional value.

Let us take the value of $\alpha_1,\alpha_2,\alpha_3$ equal too 0.9 we will approximate value of $s(t),i(t),r(t)$ as the following.

Put $K=0$ and use β_1,β_2,β_3 equal to 10 ,

$$S(0+.9\times10)=S(9)=\frac{\Gamma(1+0)}{\Gamma(0.9+1+0)}\{(1-P)\pi-\beta(0)-\pi S(0)\ \}$$

$$=\frac{1}{\Gamma(19/10)}\{(1-P)\pi-\beta.0.1-\pi.0\}=\frac{(1-P)\pi}{\Gamma(19/10)}$$

$$\mathrm{I}(0+.9\times10)=I(9)=\frac{\Gamma(1+0)}{\Gamma(0.9+1+0)}\{\beta S(0)I(0)-(\gamma+\pi)\mathrm{I}(0)\ \}=\frac{-(\gamma+\pi)}{\Gamma(19/10)}$$

$$\mathrm{R}(0+.9\times10)=R(9)=\frac{\Gamma(1+0)}{\Gamma(0.9+1+0)}\{P\pi+\gamma I(0)-\pi\,\mathrm{R}(0)\ \}=\frac{P\pi-\pi+\gamma}{\Gamma(19/10)}=\frac{\pi(\mathrm{P}-1)+\gamma}{\Gamma(19/10)}.$$

Put $K=9$

$$S(9+9)=S(18)=\frac{\Gamma(1+9/10)}{\Gamma(0.9+1+9/10)}\{(1-P)\pi-\beta S(9)I(0)-\pi S(9)\ \}$$

$$=\frac{\Gamma(19/10)}{\Gamma(28/10)}\left\{(1-p)\pi-\frac{\beta\pi(1-p)}{\Gamma(19/10)}-\frac{\pi^2(1-p)}{\Gamma(19/10)}\right\}=\frac{\pi}{\Gamma(28/10)}\{(1-p)\pi[\Gamma(19/10)-\beta-\pi]\}$$

Put $K=18$

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$$S(18+9)=S(27)=\frac{\Gamma(1+18/10)}{\Gamma(0.9+1+18/10)}\{(1-P)\pi-\beta S(18)I(0)-\pi S(18)\ \}$$

$$=\frac{\Gamma(28/10)}{\Gamma(37/10)}\left\{(1-p)\pi-\frac{\beta\pi(1-P)}{\Gamma(28/10)}[\Gamma(19/10)-\beta-\pi]-\frac{\pi^2(1-P)}{\Gamma(28/10)}[(19/10)-\beta-\pi]\right\}$$

$$=\frac{\pi(1-P)}{\Gamma(37/10)}\left\{\Gamma(28/10)-\beta(\Gamma(19/10)-\beta-\pi)-\pi^2((\Gamma(19/10)-\beta-\pi)\right\}$$

$$I(18+9)=I(27)=\frac{\Gamma(1+18/10)}{\Gamma(0.9+1+18/10)}\{\beta S(18)I(0)-(\gamma+\pi)I(18)\ \}$$

$$=\frac{\Gamma(28/10)}{\Gamma(37/10)}\left\{(\frac{\beta\pi(1-p)}{\Gamma(28/10)}(\Gamma(19/10)-\beta-\pi)-\frac{(\gamma+\pi)}{\Gamma(28/10)}(\beta\pi(1-P)+(\gamma+\pi)^2)\right\}$$

$$=\frac{1}{\Gamma(37/10)}\left\{(\beta\pi(1-p)(\Gamma(19/10)-\beta-\pi)-(\gamma+\pi)(\beta\pi(1-P)+(\gamma+\pi)^2)\right\}.$$

$$R(18+9)=R(27)=\frac{\Gamma(1+18/10)}{\Gamma(0.9+1+18/10)}\{P\pi+\gamma I(18)-\pi R(18)\ \}$$

$$=\frac{\Gamma(28/10)}{\Gamma(37/10)}\left\{P\pi-\frac{\gamma}{\Gamma(28/10)}(\beta\pi(1-P)+(\gamma+\pi)^2)-\frac{\pi}{\Gamma(28/10)}(\Gamma(19/10)P\pi-\gamma(\gamma+2\pi)-\pi^2(P-1)\right\}$$

$$=\frac{1}{\Gamma(37/10)}\left\{\Gamma(28/10)P\pi-\gamma(\beta\pi(1-P)+(\gamma+\pi)^2)-\pi(\Gamma(19/10)P\pi-\gamma(\gamma+2\pi)-\pi^2(P-1)\right\}$$

So we have got the solution as the following

$$S(t)=\frac{(1-P)\pi}{\Gamma(19/10)}t^{9/10}+\frac{\pi}{\Gamma(28/10)}\{(1-p)[\Gamma(19/10)-\beta-\pi]\}t^{18/10}+\frac{\pi(1-P)}{\Gamma(37/10)}\left\{\Gamma(28/10)-\beta(\Gamma(19/10)-\beta-\pi)-\pi^2(\Gamma(19/10)-\beta-\pi)\right\}t^{27/10}+.......$$

$$I(t)=1-\frac{(\gamma+\pi)}{\Gamma(19/10)}t^{9/10}+\frac{1}{\Gamma(28/10)}\left\{(\beta\pi(1-p)+(\gamma+\pi)^2\right\}t^{18/10}+\frac{1}{\Gamma(37/10)}\left\{(\beta\pi(1-p)(\Gamma(19/10)-\beta-\pi)-(\gamma+\pi)(\beta\pi(1-P)+(\gamma+\pi)^2)\right\}t^{27/10}+.......$$

$$R(t)=1+\frac{\pi(P-1)+\gamma}{\Gamma(19/10)}t^{9/10}+\frac{1}{\Gamma(28/10)}\left\{P\pi\Gamma(19/10)-\gamma(\gamma+2\pi)-\pi^2(p-1)\right\}t^{18/10}$$

$$+\frac{1}{\Gamma(37/10)}\left\{\Gamma(28/10)P\pi-\gamma(\beta\pi(1-P)+(\gamma+\pi)^2)-\pi(\Gamma(19/10)P\pi-\gamma(\gamma+2\pi)-\pi^2(P-1)\right\}t^{27/10}+....$$

The following table showing the results:

OBVIOUSLY TABLE 2

K	S(K)	I(K)	R(K)
0	$\frac{(1-P)\pi}{\Gamma(19/10)}$	$\frac{(\gamma+\pi)}{\Gamma(19/10)}$	$\frac{\pi(P-1)+\gamma}{\Gamma(19/10)}$
9	$\frac{\pi}{\Gamma(28/10)}\{(1-p)[\Gamma(19/10)-\beta-\pi]\}$	$\frac{1}{\Gamma(28/10)}\left\{(\beta\pi(1-p)+(\gamma+\pi)^2\right\}$	$\frac{1}{\Gamma(28/10)}\left\{P\pi\Gamma(19/10)-\gamma(\gamma+2\pi)-\pi^2(p-1)\right\}$

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18	$\frac{\pi(1-P)}{\Gamma(37/10)}\begin{Bmatrix}\Gamma(28/10)-\beta(\Gamma(19/10)-\beta-\pi)\\ -\pi^2(\Gamma(19/10)-\beta-\pi)\end{Bmatrix}$	$\frac{1}{\Gamma(37/10)}\begin{Bmatrix}(\beta\pi(1-p)(\Gamma(19/10)-\beta-\pi)\\ -(\gamma+\pi)(\beta\pi(1-P)+(\gamma+\pi)^2)\end{Bmatrix}$	$\frac{1}{\Gamma(37/10)}\begin{Bmatrix}\Gamma(28/10)P\pi-\gamma(\beta\pi(1-P)+(\gamma+\pi)^2)\\ -\pi(\Gamma(19/10)P\pi-\gamma(\gamma+2\pi)-\pi^2(P-1)\end{Bmatrix}$

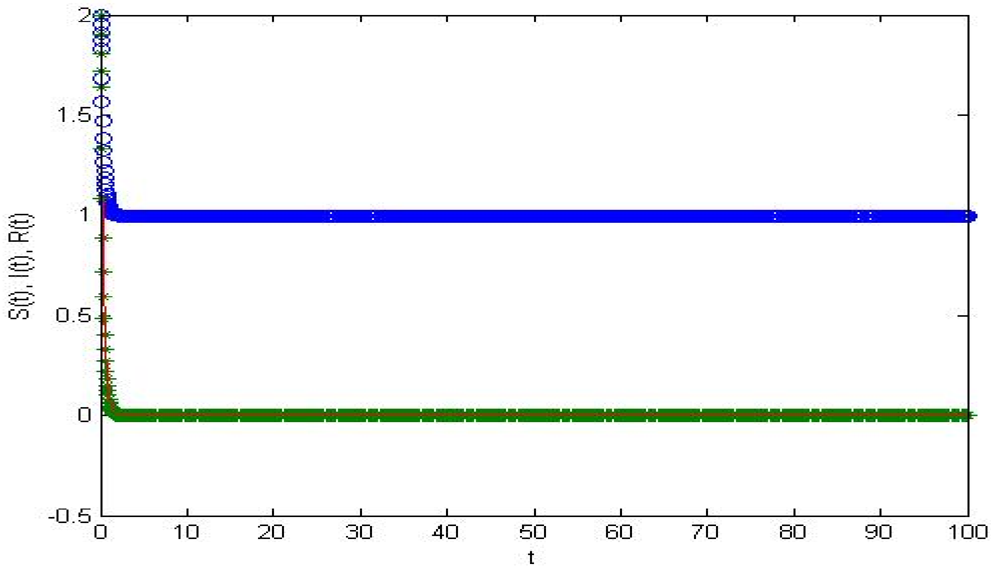

Figure 1. The behavior of the approximate solution ($S_m(t)$, $I_m(t)$, and $R_m(t)$) respectively, at $\propto = 0.9$, $\beta = 0.001$, $\gamma = 0.25$, with initial conditions S(0)=5000, I(0)=5, R(0)=0.

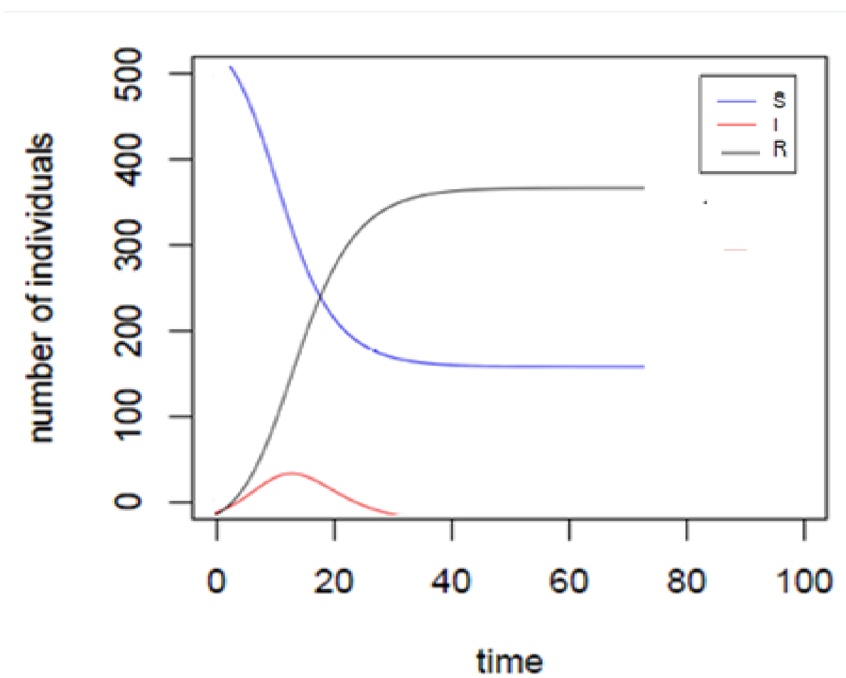

Figure 2. The 4-order Rung-Kutta method for SIR model with $dt = 0.1$, $\beta = 0.001$, $\gamma = 0.25$, with the same initial conditions of the approximate solution ($S_m(t)$, $I_m(t)$, and R_m (t)) respectively, at $\propto = 0.9$, $\beta = 0.001$, $\gamma = 0.25$, with initial conditions S(0)=5000, I(0)=5, R(0)=0.

From these figures, we can confirm that the approximate solution is an excellent agreement with the solution using the fourth order Runge-Kutta. Also, from the figures, we can conclude that the behavior of the approximate solution depends on the order of the fractional derivative.

5. CONCLUSION

In this article, we have used a new solution of SIR model by using the differential fractional transformation method to study the effect of the vaccine on diseases**.** We have used some definitions such as Aids Caputo definition of fractional calculus, as well as the definition of the Riemann-Liouville. Also we have provided some scientific theories in ways to solve differential equations which fractional contributed significantly in solving the model sports SIR, It may be concluded that this technique is very powerful.

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