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SOME CHARACTERIZATIONS FOR KONOIDAL RULED SURFACES IN DUAL SPACE

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ABSTRACT

In the paper, konoidal, strong konoidal, and orthokonoidal ruled surfaces, defined in [9], are studied in dual space, and then some characterizations of their surfaces are given.

KEYWORDS: Konoidal ruled surface, dual space.

I.INTRODUCTION AND PRELIMINARIES

Definition I.1 The set ID = {A = $a, a^* \mid a + \varepsilon a^* \in IR, \varepsilon^2=0$ } is called the dual numbers set.

Definition I.2 The set K = { $\vec{X} = \vec{x} + \varepsilon\overrightarrow{x^*} \mid \parallel \vec{X} \parallel = (1,0), \vec{x}, \overrightarrow{x^*} \in IR^3$ } is called the dual unit sphere.

Theorem I.1 (E.STUDY) The oriented lines in IR^3 are in one to one correspondence with the points of the dual unit sphere.

According to this theorem, the dual unit vector $\vec{A}=\vec{a}+\varepsilon\overrightarrow{a^*}$ is determined by means of a unique directed line in IR^3 where $\vec{a}$ and $\overrightarrow{a^*}$ are the direction vector of the line and the vectorel moment of the unit vector $\vec{a}$ with respect to the origin, respectively.

Definition I.3 $\phi = \varphi + \varepsilon\varphi^*$ being the dual angle between $\vec{A}$ and $\vec{B}$, and we have

$$< \vec{A} , \vec{B} > = \cos(\varphi + \varepsilon\varphi^*) = \cos\phi.$$

We consider two dual unit spheres K (moving) and K′ (fixed) in ID^3. Let 0 be the common center and two orthonormal dual coordinate systems $\{\overrightarrow{U_1}, \overrightarrow{U_2}, \overrightarrow{U_3}\}$ and $\{\overrightarrow{V_1}, \overrightarrow{V_2}, \overrightarrow{V_3}\}$ be rigidly linked to the spheres K and K′, respectively. For the coordinate systems, there is a linear transformation

$$U = AE$$

where A = [a_{ij} (t)+ ${a^*}_{ij}$ (t)], t ∈ IR, is a pure orthogonal matrix. If the transformation can be differentiated with respect to the parameter t, the transformation is called one parameter dual spherical motion of K relative to K′ and will be denoted by K/K′. If A(t) is a periodic matrix, the motion is called one parameter dual closed spherical motion.

Definition I.4 Let the closed ruled surface drawing by the first axis of the moving system in the dual spherical motion K/K′ denote $(\overrightarrow{U_1})$=$(\overrightarrow{U_1}(t))$, t ∈ IR. Let us take the dual unit vector $\overrightarrow{N_1}=\cos\phi\,\overrightarrow{U_2}+\sin\phi\,\overrightarrow{U_3}$ which made the dual angle $\phi(t) = \varphi(t) + \varepsilon\varphi^*(t)$ with the dual vector $\overrightarrow{U_2}$ in dual plane $(\overrightarrow{U_2}, \overrightarrow{U_3})$. Let the line corresponding to the dual unit vector $\overrightarrow{N_1}$ draw a developable surface along a curve of the ruled surface while the unit vector $\overrightarrow{U_1}$ of the moving sphere K in the dual motion K/K′ is drawing the closed ruled surface $(\overrightarrow{U_1})$. Drawing the curve is called orthogonal orbit of the closed ruled surface $(\overrightarrow{U_1})$.

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II. GENERALIZED RULED SURFACES

$\Phi(t, u_1, u_2, \ldots, u_k) = \alpha(t) + \sum_{i=1}^{k} u_i e_i(t)$ …(1)

is a parametrization for (k+1)-dimensional generalized ruled surface in n-dimensional Euclidean space E^n. $\{e_1, e_2, \ldots, e_k\}$ being an orthonormal base (ONF) for generator space $E_k(t)$ of Φ. An orthonormal base $\{e_1, e_2, \ldots, e_k, a_{k+1}, \ldots, a_{k+m}\}$

can be found for asymptotic bundle A(t) = Sp$\{e_1, e_2, \ldots, e_k, \dot{e}_1, \dot{e}_2, \ldots, \dot{e}_k\}$. And also $\{e_1, e_2, \ldots, e_k, a_{k+1}, \ldots, a_{k+m}, a_{k+m+1}\}$ is an orthonormal base for tangential bundle T(t) = Sp$\{e_1, e_2, \ldots, e_k, a_{k+1}, \ldots, a_{k+m}, \dot{\alpha}\}$ of Φ, [9]. Here we have

dimA(t)= k+m, $0 \le m \le k$, …(2)

k+m$\le dimT(t) \le$k+m+1, …(3)

If dim T(t)=k+m, Φ has an edge space and if dim T(t)=k+m+1, Φ has a central space.

We complete the ONF $\{e_1, e_2, \ldots, e_k, a_{k+1}, \ldots, a_{k+m}\}$ of the generalized ruled surface with edge ruled surface and the ONF $\{e_1, e_2, \ldots, e_k, a_{k+1}, \ldots, a_{k+m}, a_{k+m+1}\}$ of the generalized ruled surface with central ruled surface by an arbitrary ONF $\{a_{k+m+1}, \ldots, a_n\}$ respectively $\{a_{k+m+2}, \ldots, a_n\}$ of the orthogonal complement called a complementary ONF, [8].

For $\eta_{m+1} \neq 0$, we have

$\dot{\alpha} = \sum_{i=1}^{k} \xi_i e_i + \eta_{m+1} a_{k+m+1}$. …(4)

By differentiation of elements of ONF $\{e_1, e_2, \ldots, e_k, a_{k+1}, \ldots, a_{k+m}, \ldots, a_n\}$, we have

$\dot{e}_i = \sum_{j=1}^{k} \alpha_{ij} e_j + \kappa_i a_{k+i}$, $1 \le i \le m$, $\kappa_i > 0$,…(5)

$\dot{e}_i = \sum_{j=1}^{k} \alpha_{ij} e_j$, m$< i \le k$, …(6)

$\dot{a}_{k+i} = -\kappa_i e_i + \sum_{j=1}^{m} \tau_{ij} a_{k+j} + \omega_i a_{k+m+1} + \sum_{\lambda=2}^{n-k-m} \gamma_{i\lambda} a_{k+m+\lambda}$, $1 \le i \le m$, …(7)

$\dot{a}_{k+m+1} = -\sum_{j=1}^{m} \omega_j a_{k+j} - \sum_{\lambda=2}^{n-k-m} \beta_\lambda a_{k+m+\lambda}$, …(8)

$\dot{a}_{k+m+s} = \sum_{j=1}^{m} \omega_{sj} a_{k+j} + \beta_s a_{k+m+1} + \sum_{\lambda=2}^{n-k-m} \beta_{s\lambda} a_{k+m+\lambda}$, $2 \le s \le n-k-m$, …(9)

[9].

Definition II.1 If E^n has a subspace E^q, q$\ge k$, which is constant and parallel to the generator space E_k(t) of Φ, the (k+1)-ruled surface Φ is called q- konoidal and the subspace $E^q \subset E^n$ is called directional space, [9]. Φ being q-konoidal with central ruled surface Ω, if the tangent space at the central points of Φ is orthogonal to E^q, the ruled surface Φ is called q-orthokonoidal and if q= dim A(t)=k+m= cons., Φ is called strong-konoidal, [9].

III. KONOIDAL RULED SURFACES IN DUAL SPACE

Definition III.1 Let $\{\vec{U_1}, \vec{U_2}, \vec{U_3}\}$ and $\{\vec{V_1}, \vec{V_2}, \vec{V_3}\}$ be two sets of dual orthonormal unit vectors which are rigidly linked to the moving dual sphere K, and the fixed the dual sphere K', respectively. Denote the dual fixed angle between $\vec{U_1}$ and $\vec{V_1}$ vectors, by $\Theta(t) = \theta(t) + \varepsilon\theta^*(t)$.

If we have $(\vec{V_1})$ dual ruled surface which is paralel to $(\vec{U_1})$ dual ruled surface which is drawn during the motion K/K' by $\vec{V_1}$ dual unit vector is defined as

$$\vec{V_1} = \cos\Theta\,\vec{U_1} + \sin\Theta\,\vec{U_2}$$

the 2-dimensional ruled surface $(\vec{U_1})$ is called 2-konoidal ruled surface in the dual space, and the dual ruled surface $(\vec{V_1})$ is called directional space.

Definition III.2 Let $(\vec{U_1})$ be dual closed ruled surface, and also, let $\phi(t) = \varphi(t) + \varepsilon\varphi^*(t)$, t$\in$ I, dual fixed angle between $\vec{U_2}$ and $\vec{N_1}$ which is defined as

$$\vec{N_1} = \cos\phi\,\vec{U_2} + \sin\phi\,\vec{U_3}$$

where $\vec{N_1}$ is a dual unit vector on the plane of $(\vec{U_2}, \vec{U_3})$. While $\vec{U_1}$ is drawing $(\vec{U_1})$ dual closed ruled surface, during the closed motion K/K', $\vec{N_1}$ is drawing a curve on the fixed dual sphere K'. This curve is an orthogonal orbit of the $(\vec{U_1})$ dual ruled surface.

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If the tangent space at the orthogonal orbit's points of $(\vec{U_1})$ is orthogonal to $(\vec{V_1})$, $(\vec{U_1})$ is called 2-orthokonoidal ruled surface in the dual space.

Definition III.3 If dimA(t)=m+1=const., and m=1, $(\vec{U_1})$ is called strongkonoidal ruled surface in dual space.

Corollary III.1 m=1 is always true that is given contidions in, so the ruled surface ($\vec{U_1}$) is strongkonoidal for every case.
For the leading curve X, which is drawn by $\vec{U_1}$ unit dual vector on the fixed sphere K,

$$X(t)= \xi_1 e_1+\eta_1 a_2+\eta_2 a_3,\ \eta_1 \neq 0,\ \eta_2 \neq 0$$

where { e_1(t) , a_2(t) } is an ONF of the tangent bundle T(t) of ($\vec{U_1}$) . By differentiation of elements of ONF { e_1 (t) , a_2 (t) , a_3 (t)}, we recalculated them for dual space forms

$$(10)\ldots \begin{cases} \dot{e_1} = \alpha\, e_1 + \kappa_1 a_2, & \kappa_1 > 0, \\ \dot{a}_2 = -\kappa_1 e_1 + \tau_1 a_2 + \omega_1 a_3, & \\ \dot{a}_3 = -\kappa_2 e_2 + \tau_2 a_2 + \omega_2 a_3, \kappa_2 > 0 & \end{cases}$$

where

$$\kappa_1 = k_1 + \varepsilon k^*{}_1,\quad \kappa_2 = k_2 + \varepsilon k^*{}_2,$$

$$\tau_1 = t_1 + \varepsilon t^*{}_1,\quad \tau_2 = t_2 + \varepsilon t^*{}_2,$$

$$\omega_1 = v_1 + \varepsilon v^*{}_1,\quad \omega_2 = v_2 + \varepsilon v^*{}_2.$$

Theorem III.1 Let $(\vec{U_1})$ be 2-ruled surface and dimA(t) be constant. The asymptotic bundle A(t) of $(\vec{U_1})$ is paralel to directional space $(\vec{V_1})$.

Proof. Let $(V_1)^\perp$ be orthogonal complement of $(\vec{V_1})$. Since $\dim(V_1)^\perp$=1, so we can take

$$(V_1)^\perp = \mathrm{Sp}\,\{b_3(t)\}.$$

Because of $(V_1)^\perp$ is orthogonal to $(\vec{V_1})$

$$< b_3, e_1(t) > = 0 \qquad \ldots(11)$$

From (10) by differentiation we get

$$< b_3, \dot{e_1}(t) > = 0 \qquad \ldots(12)$$

Let us rewrite (10) and use (11). After some algebraic operation we get

$$< b_3\,,\ \alpha\, e_1 + \kappa_1 a_2 > = 0$$

$$< b_3, a_2(t) > = 0 \qquad \ldots(13)$$

From (11) we obtain the asymptotic bundle A(t) of $(\vec{U_1})$ is orthogonal to $(V_1)^\perp$, and hence

A(t) ∥ $(\vec{V_1})$.

Theorem III.2 Let $(\vec{U_1})$ be noncylindiric 2-konoidal ruled surface. If ($\vec{U_1}$) is strongkonoidal (in the equations (10)),

$$w_1 = 0.$$

Proof. From A(t) ∥ $(\vec{V_1})$ we get ($\forall t \in I$)

$$< b_3, a_2(t) > = 0$$

Some algebraic operation we get

$$< b_3, \dot{a}_2(t) > = 0$$

$$< b_3, -\kappa_1 e_1 + \tau_1 a_2 + \omega_1 a_3 > = 0$$

$$\omega_1 < b_3, a_3 > = 0 \qquad \ldots(14)$$

Now ($\vec{U_1}$) be strongkonoidal. In this instance directional space $(\vec{V_1})$ is stretched by base vectors of A(t). Let $(V_1)^\perp$ = Sp $\{b_3\}$ be orthogonal complement of $(\vec{V_1})$. Then $(V_1)^\perp$ and Sp $\{a_3\}$denote the same space. Then, from (14) we obtain

$$\omega_1 = 0.$$

Theorem III.3 Let 2-ruled surface ($\vec{U_1}$) be with central ruled surface. If $(\vec{U_1})$ is strongkonoidal, then $(\vec{U_1})$ is orthokonoidal.

Proof. We know that A(t) is paralel to $(\vec{V_1})$. And the tangent space at the orthogonal orbit points of $(\vec{U_1})$ are orthogonal to A(t), and also are orthogonal to $(\vec{V_1})$.
And then, from definition of orthokonoidal, we obtain that $(\vec{U_1})$ is orthokonidal.

Theorem III.4 Suppose that $(\vec{U_1})$ be noncylindiric and strongkonoidal 2-ruled surface. If ($\vec{U_1}$) is orthokonoidal, then in (10)

$$\omega_2 = 0.$$

Proof. Let $(\overrightarrow{U_1})$ be strongkonoidal. Then m=1. And so $(\overrightarrow{U_1})$ is strongkonoidal, it has central ruled surface and $\overrightarrow{a_3}$(t) , t∈ I, central tangent vector.

Now, let $(\overrightarrow{U_1})$ be orthokonoidal. Because of

$(V_1)^{\perp}$ = Sp $\{b_3\}$, $\overrightarrow{a_3} \perp \overrightarrow{V_1}$. Then

$$< b_3, a_3(t) > = \lambda, \quad (\lambda\ const.)$$

After some algebraic operation we get

$$< b_3, \dot{a}_3(t) > = 0$$

$$< b_3, -\kappa_2 e_2 + \tau_2 a_2 + \omega_2 a_3 > = 0$$

$$\omega_2 < b_3, a_3 > = 0$$

From Theorem III.2 we obtain

$$\omega_2 = 0.$$

Theorem III.5 Supose that 2-dual ruled surface $(\overrightarrow{U_1})$ has central ruled surface. If $(\overrightarrow{U_1})$ is 2-orthokonoidal, then

$$\omega_1 = 0.$$

Proof. Since $(\overrightarrow{U_1})$ has central ruled surface, İt has also $\overrightarrow{a_3}$(t) central tangent vector.

Let $(\overrightarrow{U_1})$ be 2- orthokonoidal. Then $\overrightarrow{a_3} \perp \overrightarrow{V_1}$, that is, $\overrightarrow{a_3} \in (V_1)^{\perp}$.

Some algebraic operation we get

$$< a_3(t_0), a_2(t) > = 0$$

$$< a_3(t_0), \dot{a}_2(t) > = 0$$

$$< a_3(t_0), -\kappa_1 e_1 + \tau_1 a_2 + \omega_1 a_3 > = 0$$

and consequently we obtain

$$\omega_1 = 0.$$

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