March, 2014. Vol. 4, No. 10 ISSN2305-8269
International Journal of Engineering and Applied Sciences

www.eaas-journal.org

FUZZY GRAPH COLORING USING α CUTS

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Abstract

In this paper we define the fuzzy chromatic number, chromatic index and fuzzy total chromatic number of a fuzzy graph as fuzzy numbers through the α- cuts of the fuzzy graph which are crisp graphs. We explain these concepts through examples.

Key words: Chromatic number, Chromatic index, total chromatic number,Fuzzy set,α cut.

1. Introduction:

Many problems of practical interest that can be modeled as graph theoretic problems may be uncertain. To deal with this uncertainty the concept of fuzzy theory was applied to graph theory.

In 1965 , Zadeh introduced the notion of fuzzy set which is characterized by a membership function which assigns to each object a grade of membership which ranges from 0 to 1. The first definition of fuzzy graph was introduced by Kaufmann (1973), based on Zadeh's fuzzy relations (1971). As explained in[5] fuzzy graphs may be defined by considering fuzzy set of crisp graphs or fuzzy edge set with crisp vertex set or fuzzy vertex set with crisp edge set or fuzzy vertex and edge set or crisp vertices and edges with fuzzy connectivity or crisp graph with fuzzy weights. In this paper we consider the fuzzy graphs with crisp vertex set and fuzzy edge set.

Coloring of graphs is a most important concept in which we partition the vertex (edge) set of any associated graph so that adjacent vertices (edges) belong to different sets of the partitions .In other words coloring problem is considered as grouping the items of interest as few groups as possible so that incompatible items are in different groups.

Generally for a given graph G=(V,E) , a coloring function is a mapping $\pi : V\rightarrow N$ such that $\pi(i) \neq \pi(j)$ where i and j are adjacent vertices in G(incompatible vertices)but if we use only k colors to color a graph we define a k-coloring $\pi^k : V\rightarrow\{1,2,\dots.k\}$. A graph is k-colorable if it admits a k- coloring. The chromatic number $\chi(G)$,of a graph G is the minimum k for which G is k- colorable.

Fuzzy graph coloring is one of the most important problems of fuzzy graph theory; it is mainly studied in combinatorial optimization like traffic light control, exam scheduling, register allocation etc.

Definition 1.1:

A fuzzy set A defined on a non empty set X is the family $A=\{(x,\mu_A(x))/ x \in X\}$ where $\mu_A: X\rightarrow I$ is the membership function. In classical fuzzy set theory the set I is usually defined on the interval [0,1] such that $\mu_A(x) = 0$ if x does not belong to A $\mu_A(x) = 1$ if x strictly belongs to A and any intermediate value represents the degree in which x could belong to A. the set I could be discrete set of the form $I=\{0,1,\dots..k\}$ where $\mu_A(x) < \mu_A(x')$ indicates that the degree of membership of x to A is lower than the degree of membership of x'.

2.Fuzzy graphs with crisp vertices and fuzzy edges:

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Definition 2.1:

The graph $\hat{G} = (V,\hat{E})$ is a fuzzy graph where V is the vertex set and the fuzzy edge set is characterized by the matrix $\mu = [\mu_{ij}]_{i,j\in V}$ $\mu_{ij} = \mu_{\hat{E}}(\{i,j\})$ for every $i,j \in V$ such that $i \neq j$ and $\mu_{\hat{E}} : VXV \rightarrow I$ is the membership function.

Each element $\mu_{ij} \in$I represents the intensity level of the edge $\{i,j\}$ for any $i,j \in V$ with $i \neq j$. The fuzzy graph can also denoted by $\hat{G}=(V,\mu)$.

The set I is linearly ordered in such a way that the expression $\mu_{ij} < \mu_{i'j'}$ stands for "the intensity level of edge $\{i,j\}$ is lower than the intensity level of edge$\{i',j'\}$". The fuzzy graph $\hat{G}$ can be considered as generalization of the crisp (incompatibility) graph G, since, taking I $= \{0,1\}$, $\hat{G}$ becomes a crisp graph.

3.Fuzzy vertex coloring:[3,6]

Definition 3.1:

A fuzzy set A defined on X can be characterized from its family of α-cuts $A_\alpha = \{x \in X/ \ \mu_A(x) \geq \alpha\}$ $\alpha \in$I. This family of sets is monotone, i.e., for $\alpha,\beta \in$I $\alpha \leq \beta$ we have $A_\alpha \supseteq A_\beta$

On the other hand, given a finite monotone family $\{ A_\alpha^p / p \in \{1,....m\}\}$, a fuzzy set can be defined from the membership function

$\mu_A(x) = sup\{ \alpha^P \ / x \in A_\alpha^p \}$ for every $x \in X$.

Let $\{G_\alpha = (V,E_\alpha)/ \alpha \in I\}$ be the family of α-cuts of $\hat{G}$, where the α-cut of a fuzzy graph is the crisp graph $G_\alpha = (V,E_\alpha)$ with $E_\alpha = \{\{i,j\}/i,j \in V, \mu_{ij} \geq \alpha\}$.

Hence any crisp k-coloring π^k_α can be defined on G_α. The k-coloring function of $\hat{G}$ is defined through this sequence.

For each $\alpha \in I$, let χ_α denote the chromatic number of G_α.The chromatic number of $\hat{G}$ is defined through a monotone family of sets.

Definition 3.2 :

For a fuzzy graph $\hat{G} = (V,\mu)$, its chromatic number is the fuzzy number $\chi(\hat{G}) = \{x,v(x)) \ / \ x \in X\}$, where $X = \{1,...|V|\}$, $v(x) = sup \ \{\alpha \in I \ / x \in A_\alpha\}$ $x \in X$ and $A_\alpha = \{1,...\chi_\alpha\}$ $\alpha \in I$.

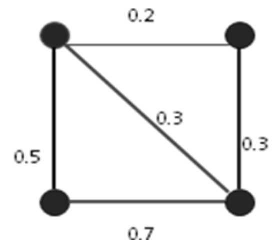
The chromatic number of a fuzzy graph is a normalized fuzzy number whose modal value is associated with the empty edge-set graph. Its meaning depends on the sense of index α. It can be interpreted that for lower values of α there are many incompatible edges between the vertices so that more colors are needed in order to consider the incompatibilities; on the other hand, for higher values of α there are fewer incompatible edges and less colors are needed. The fuzzy coloring problem consists of determining the chromatic number of a fuzzy graph and an associated coloring function.

For any level α, the minimum number of colors needed to color the crisp graph G_α will be computed. In this way the fuzzy chromatic number is defined as fuzzy number through its α-cuts.

Example: 1

Consider the fuzzy graph $\hat{G} = (V,\mu)$ *where* $V = \{1,2,3,4\}$ and the matrix of μ is defined as

$$\mu = \begin{bmatrix} 0 & 0.2 & 0.3 & 0.5 \\ 0.2 & 0 & 0.3 & 0 \\ 0.3 & 0.3 & 0 & 0.7 \\ 0.5 & 0 & 0.7 & 0 \end{bmatrix}$$



For this example, six crisp graphs $G_\alpha=(V,E_\alpha)$ are obtained by considering the values $\alpha \in$I . For each $\alpha \in$ I, the table 2.1 contains the edge set E_α , the chromatic number χ_α and a χ_α – coloring $\pi^{\chi_\alpha}_\alpha$

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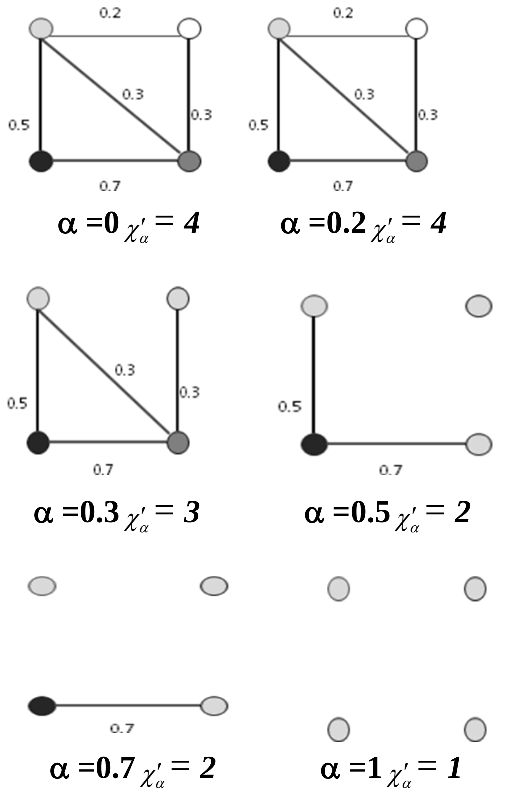

Table 3.1

The fuzzy chromatic number of Ĝ is

χ(Ĝ)={ (1,1),(2,0.7),(3,0.3),(4,0.2)}

4.Fuzzy edge coloring:

Let $\{G_\alpha = (V,E_\alpha)/ \alpha\in I\}$ be the family of α-cuts of Ĝ, where the α-cut of a fuzzy graph is the crisp graph $G_\alpha = (V,E_\alpha)$ with $E_\alpha = \{\{i,j\}/i,j\in V,\mu_{ij}\geq\alpha\}$.

In crisp case the edge chromatic number of a graph is either *Δ* or *Δ+1* where Δ is the maximum vertex degree.

Here we define fuzzy edge chromatic number as a fuzzy number as follows:

Definition 4.1:

For a fuzzy graph $\hat{G} = (V,\mu)$, its edge chromatic number is the fuzzy number $\chi'_f(\hat{G}) = \{x, \lambda(x)) / x\in X\}$, where $X = \{1,\dots \Delta+1\}$, $\lambda(x) = \sup\{\alpha\in I / x\in A_\alpha\}$ and $A_\alpha = \{1,\dots \chi'_\alpha\}$ $\alpha\in I$.

α	E_α	χ	$c_\alpha(12)$	$c_\alpha(13)$	$c_\alpha(14)$	$c_\alpha(23)$	$c_\alpha(34)$
0	12,13, 14,23, 24	4	1	2	3	3	1
0.2	12,13, 14,23, 24	4	1	2	3	3	1
0.3	13,14, 23, 24	3	0	2	1	1	3
0.5	14,34	2	0	0	1	0	2
0.7	34	2	0	0	0	0	1
1	Φ	1	0	0	0	0	0

*F*or example -1, six crisp graphs $G_\alpha=(V,E_\alpha)$ are obtained by considering the values α∈I . For each α∈ I, the following table -4.1 contains the edge set E_α , the edge chromatic number χ'_α

Table 4.1

ISSN2305-8269
International Journal of Engineering and Applied Sciences

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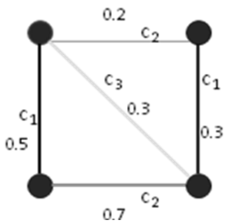
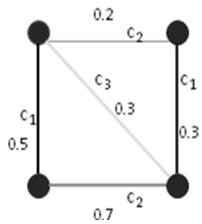
Six crisp graphs $G_\alpha = (V,E_\alpha)$ corresponding to each α is given in figure 4.1

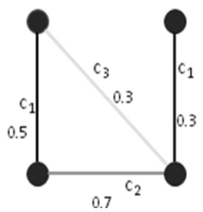
Figure-4.1

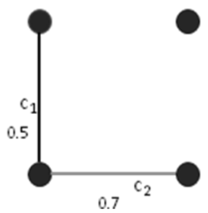
α	E_α	χ	$c_\alpha(12)$	$c_\alpha(13)$	$c_\alpha(14)$	$c_\alpha(23)$	$c_\alpha(34)$
0	12,13, 14,23, 24	3	1	2	3	3	1
0.2	12,13, 14,23, 24	3	1	2	3	3	1
0.3	13,14, 23, 24	3	0	2	1	1	3
0.5	14,34	2	0	0	1	0	2
0.7	34	1	0	0	0	0	1
1	Φ	0	0	0	0	0	0



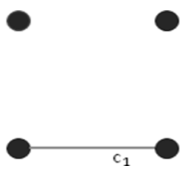


α =0 $\chi'_\alpha = 3$ **α =0.2** $\chi'_\alpha = 3$





α =0.3 $\chi'_\alpha = 3$ **α =0.5** $\chi'_\alpha = 2$



α =0.7 $\chi'_\alpha = 1$ **α =1** $\chi'_\alpha = 0$

Since Δ=3 for the given graph the fuzzy edge chromatic number of Ĝ is χ'_α ={(1,0.7),(2,0.5),(3,0.3),(4,0)}.

Remark:

If μ =1 for all edges of the graph Ĝ then it becomes the crisp graph G then $\chi'_\alpha = \chi'$.

5.Fuzzy Total coloring:

Let $\{G_\alpha = (V,E_\alpha)/\ \alpha \in I\}$ be the family of α-cuts of Ĝ, where the α-cut of a fuzzy graph is the crisp graph $G_\alpha = (V,E_\alpha)$ with $E_\alpha = \{\{i,j\}/i,j \in V, \mu_{ij} \geq \alpha\}$.

In crisp case the total chromatic number of a graph is atmost *Δ+2* *(By total coloring conjecture)* where Δ is the maximum vertex degree.

So we define fuzzy total chromatic number as a fuzzy number as follows:

Definition 5.1 :

For a fuzzy graph $\hat{G} = (V,\mu)$, its total chromatic number is the fuzzy number $\chi_f^T(\hat{G})$ $= \{x,\ \tau(x)\ /\ x \in X\}$, where $X = \{1,\ldots\ \Delta+2\}$, $\tau(x) = \sup\{\alpha \in I\ /x \in A_\alpha\}$ and $A_\alpha = \{1,\ldots\ \chi_f^T\}\ \alpha \in I$.

Consider example: 1 For each α∈ I, the following table5.1 contains the mapping $c_\alpha: V \cup E \rightarrow \{1,2,\ldots k\}$ and the total chromatic number χ_f^T *for* G_α.

ISSN2305-8269
International Journal of Engineering and Applied Sciences

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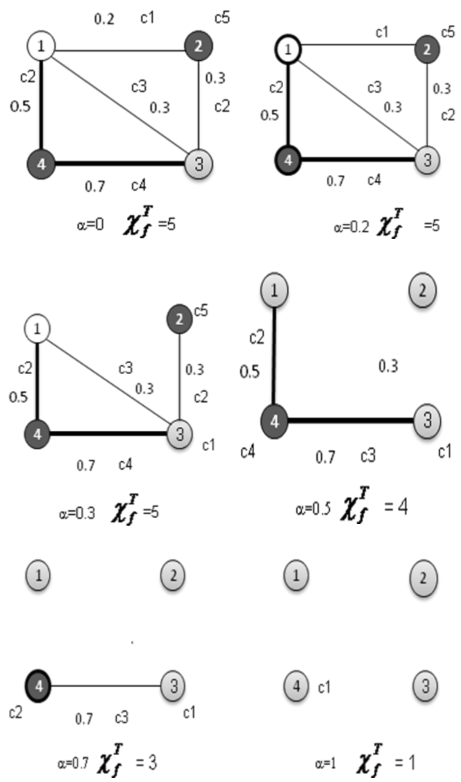
α	χ_f^T	$c_\alpha(1)$	$c_\alpha(2)$	$c_\alpha(3)$	$c_\alpha(4)$	$c_\alpha(12)$	$c_\alpha(13)$	$c_\alpha(14)$	$c_\alpha(23)$	$c_\alpha(34)$
0	5	2	5	1	5	1	4	3	3	2
0.2	5	2	5	1	5	1	4	3	3	2
0.3	5	2	1	5	1	0	4	3	3	2
0.5	4	1	1	1	2	0	0	3	0	4
0.7	3	1	1	1	3	0	0	0	0	2
1	1	1	1	1	1	0	0	0	0	0

Table 5.1

Six crisp graphs $G_\alpha = (V,E_\alpha)$ corresponding to each α is given in figure 5.1

Figure 5.1



Since Δ=3 for the given graph the fuzzy total chromatic number of Ĝ is

χ_f^T ={(1,1),(2,0),(3,0.7),(4,0.5)(5,0.3)}.

6. CONCLUSION

In this paper we defined the fuzzy chromatic number, chromatic index and fuzzy total chromatic number as fuzzy numbers through the α- cuts of the fuzzy graph which are crisp graphs.
We can also de-fuzzify this number using any of methods available if we want these numbers in crisp form.

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