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A NEW METHOD OF PARAMETRIZATION OF NEUTRINO MASS MATRIX THROUGH BREAKING OF $\mu - \tau$ SYMMETRY: NORMAL HIERARCHY.

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Abstract

The present observation data on neutrino oscillation strongly supports the deviation from Tri-Bimaximal mixing (TBM). In the first part of the present work, the $\mu - \tau$ symmetry of the neutrino mass matrix is perturbed at its minimal level in order to produce the deviation from TBM mixing. This includes nonzero value of θ_{13} along – with $tan^2\theta_{12} \neq 0.5$ and $tan^2\theta_{23} < 1$. The parametrization of the neutrino mass matrix which describes normal hierarchy (N.H) has been addressed with minimum number of independent parameters, out of which two parameters η and α control θ_{12} and θ_{13} respectively, without any interference with mass eigenvalues. In the second part, the deviation from maximal condition i.e., $\theta_{23} = \pi/4$, along-with a nonzero value of θ_{13} has been implemented with the introduction of a perturbation matrix M_s which breaks the $\mu - \tau$ symmetric mass matrix.

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Keywords: Normal Hierarchy, Tri-bimaximal mixing, neutrino masses, $\mu - \tau$ symmetry.

1. Introduction

The recent discovery that the reactor angle (θ_13) is not only non-zero but relatively large, by the Daya Bay and RENO [1,2], has a significant impact on the existing neutrino mass models. The global best-fit values of the neutrino oscillation parameters like $tan^2\theta_{12}$ and $sin^2\theta_{13}$ are 0.047 and 0.026 respectively [3]. The data of the mass squared differences are very precise and the Dirac delta phase, δ_{CP} is still in the dark. We have chosen $\delta_{CP} = 0$, through out the calculation and assumed that there is no sterile neutrino.

Many theories predict that the atmospheric mixing angle θ_{23} must depart from maximal condition [4,5,6] when $\mu - \tau$ symmetry is broken in order to produce nonzero θ_{13}. Two possibilities are there in connection with the ceviation from TBM along with the generation of non-zero θ_{13}. They are either with $\theta_{23} = \pi/4$ or with$\theta_{23} \neq \pi/ 4$. From theoretical point of view the problem can be addressed either by disturbing the $\mu - \tau$ symmetry of the neutrino mass matrix or by starting from a new PMNS matrix which can produce the present experimental results [8].

The pattern of the absolute neutrino masses whether normal (NH) or inverted (IH) is still an open question. Besides, the status of the quasi degenerate (QD) model is not yet been ruled out. The $\mu - \tau$ symmetry is capable of producing TBM mixing and can control the solar angle,θ_{12} [10, 11]. In the present work, a new method of parametrization is presented with a hope to perturb the $\mu - \tau$ symmetry as well.

$\mu - \tau$ symmetry is a very beautiful symmetry and provides a good control over the solar angle. TBM mixing is associated with symmetry groups like A_4 and S_4 and is a special case of $\mu - \tau$ symmetry. A

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$\mu - \tau$ symmetric neutrino mass matrix takes the form,

$$M_{\mu\tau} = \begin{pmatrix} A & B & B \\ B & C & D \\ B & D & C \end{pmatrix}. \qquad (1)$$

And also we get

$$tan2\theta_{12} = 2\frac{\sqrt{2}B}{A-C-D}. \qquad (2)$$

$M_{\mu\tau}$ gives $\theta_{13} = 0$ and $\theta_{23} = \pi/4$. The corresponding PMNS mixing matrix U becomes,

$$U = \begin{pmatrix} cos\theta_{12} & -sin\theta_{12} & 0 \\ \frac{sin\theta_{12}}{\sqrt{2}} & \frac{cos\theta_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{sin\theta_{12}}{\sqrt{2}} & \frac{cos\theta_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \qquad (3)$$

When $M_{\mu\tau}$ in eq (1) is broken down to a general matrix which can impart a nonzero θ_{13} (but still keeps $\theta_{23} = \pi/4$), the mass matrix and the PMNS matrix take the following structure.

$$M = \begin{pmatrix} A' & B' & B'' \\ B' & C' & D' \\ B'' & D' & C'' \end{pmatrix}, \qquad (4)$$

$$U = \begin{pmatrix} cos\theta_{12}cos\theta_{13} & -sin\theta_{12}cos\theta_{13} & sin\theta_{13} \\ \frac{1}{\sqrt{2}}(sin\theta_{12} + cos\theta_{12}sin\theta_{13}) & \frac{1}{\sqrt{2}}(cos\theta_{12} - sin\theta_{12}sin\theta_{13}) & -\frac{1}{\sqrt{2}}cos\theta_{13} \\ \frac{1}{\sqrt{2}}(sin\theta_{12} - cos\theta_{12}sin\theta_{13}) & \frac{1}{\sqrt{2}}(cos\theta_{12} + sin\theta_{12}sin\theta_{13}) & \frac{1}{\sqrt{2}}cos\theta_{13} \end{pmatrix} \qquad (5)$$

On diagonalizing M in eq (4), with this new$U$, $U^{\dagger}M\,U \rightarrow M^{diag}$, we arrive at the two important conditions, under the fulfilment of which the complete diagonalization is possible. They are,

$$tan2\theta_{12} = \frac{-4\{\sqrt{2}(B'+B'')\,cos\theta_{13} + (C'-C'')sin\;\theta_{13}\}}{2\sqrt{2}\,(B'+B'')sin2\theta_{13} + (C'+C''-2A-2D')cos2\theta_{13} - 2A' + C' + C'' + 6D'} \qquad (6)$$

And

$$cot\theta_{12} = \frac{2\{(C''-C')cos\theta_{13} + (B'+B'')sin\theta_{13}\}}{2\sqrt{2}(B'-B'')cos\theta_{13} + (C'+C''-2\,A'-2D')sin2\theta_{13}} \qquad (7)$$

These two equations involving θ_{12} and θ_{13} add little complicacy in the process of parametrization.

2. The neutrino mass model with normal hierarchy (NH).

Normal hierarchy is the case when we take absolute masses of the three neutrinos in the order, $m_1 < m_2 < m_3$. The mass m_1 is considered to be very small in comparison to m_2 and m_3 and can be taken to be nearly zero. For parametrization of the mass matrix it is always kept in mind that when the perturbation is nullified the neutrino mass matrix arrives at the original $\mu - \tau$ symmetric structure. The parametrization of the neutrino mass matrices both with $\theta_{13} = 0$ and $\theta_{13} \neq 0$, are addressed with equal footing.

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2.1 Parametrization of neutrino mass matrix with exact $\mu-\tau$ symmetric mass matrix ($\theta_{13}=0$).

We start with a $\mu-\tau$ symmetric mass matrix $M^0{}_{\mu\tau}$ which is capable of producing one of its eigenvalue equal to zero and other being unity out of the three..

$$M^0{}_{\mu\tau}=\begin{pmatrix} A & -\sqrt{Ax} & -\sqrt{Ax} \\ -\sqrt{Ax} & x+\frac{1}{2} & x-\frac{1}{2} \\ -\sqrt{Ax} & x-\frac{1}{2} & x+\frac{1}{2} \end{pmatrix}$$

Also eq (2) becomes, $$tan2\theta_{12}=2\frac{\sqrt{2Ax}}{2x-A}$$

We then choose A, x as functions of two parameters as function of ξ and η and under certain proper choice of these two functions, we formulate the following $\mu-\tau$ symmetric neutrino mass matrix which follows normal hierarchy.

$$M_{\mu\tau}=\begin{pmatrix} \xi\eta^2 & -\xi\eta\sqrt{\frac{1-\eta^2}{2}} & -\xi\eta\sqrt{\frac{1-\eta^2}{2}} \\ -\xi\eta\sqrt{\frac{1-\eta^2}{2}} & \frac{1}{2}\{1+\xi(1-\eta^2)\} & \frac{1}{2}\{\xi(1-\eta^2)-1\} \\ -\xi\eta\sqrt{\frac{1-\eta^2}{2}} & \frac{1}{2}\{\xi(1-\eta^2)-1\} & \frac{1}{2}\{1+\xi(1-\eta^2)\} \end{pmatrix} m_0 \qquad (8)$$

The eigenvalues of $M_{\mu\tau}$ in eq (8) are identified with the three absolute mass eigenvalues of the neutrinos. They are as follows.

$$m_1=0, \quad m_2=m_0\xi \quad \text{and} \quad m_3=m_0 \qquad (9)$$

Also $M_{\mu\tau}$ in eq (8) results in

$$tan2\theta_{12}=\frac{2\,\eta\sqrt{1-\eta^2}}{1-2\eta^2} \qquad (10)$$

Here ξ and η are two independent parameters and m_0 is the input. From eq (9) and (10) it is clear that the prediction of θ_{12} and the masses independently depends upon η and ξ and never interfere.

2.2 Parametrization of the neutrino mass matrix with broken $\mu-\tau$ symmetry (for $\theta_{13}\neq 0$).

We begin with a $\mu-\tau$ symmetric mass matrix $M^1_{\mu\tau}$ and perturb this with a matrix M_s. Where

$$M^1_{\mu\tau}=\begin{pmatrix} A+a & bB & bB \\ bB & C-\frac{a}{2} & D+\frac{a}{2} \\ bB & D+\frac{a}{2} & C-\frac{a}{2} \end{pmatrix} \quad \text{and} \quad M_s=\begin{pmatrix} 0 & -y & +y \\ -y & -x & 0 \\ +y & 0 & +x \end{pmatrix}. \qquad (11)$$

Now the mass matrix with broken $\mu-\tau$ symmetry becomes , $M=M^1_{\mu\tau}+M_s$. We consider $A,B,C,D,a,b,,x$ and y as functions of ξ,η and in addition one extra independent parameter α. We have to choose the a,b,x,y in such a way that under the choice of $\alpha=0$, M must coincide with $M_{\mu\tau}$ in eq (8). This condition allows us to

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choose $A = \xi\eta^2$, $B = -\xi\eta\sqrt{(1-\eta^2)/2}$, $C = 1/2\ \{1+\xi(1-\eta^2)\}$ and $D = 1/2\ \{\xi(1-\eta^2)-1\}$. We assume the structure of a, b as $a = \alpha^2(1-\xi\eta^2)$ and $b = \sqrt{1-\alpha^2}$. M must follow the eigenvalue equation, $det|M-\lambda_i| = 0$, where λ_i are the three eigen values of M. We expect as before one of the eigenvalues to be unity and the other to be zero. i.e. there are two equations, $det|M| = 0$ and $det|M-I| = 0$. solving which we work out the texture of x and y. Finally we obtain,

$$M = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{12} & m_{22} & m_{23} \\ m_{13} & m_{23} & m_{33} \end{pmatrix} m_0 \qquad (12)$$

Where,

$$m_{11} = \xi\eta^2 + (1-\xi\eta^2)\alpha^2,$$

$$m_{12} = -\sqrt{\frac{1-\alpha^2}{2}}\left\{\xi\eta\sqrt{1-\eta^2} + \alpha(1-\xi\eta^2)\right\},$$

$$m_{13} = -\sqrt{\frac{1-\alpha^2}{2}}\left\{\xi\eta\sqrt{1-\eta^2} - \alpha(1-\xi\eta^2)\right\},$$

$$m_{22} = \frac{1}{2}\left\{1-\alpha^2+\xi\left(\sqrt{1-\eta^2}-\alpha\eta\right)^2\right\},$$

$$m_{23} = \frac{1}{2}\{\xi(1-\eta^2)+\alpha^2(1-\xi\eta^2)-1\},$$

$$m_{33} = \frac{1}{2}\left\{1-\alpha^2+\xi\left(\alpha\eta+\sqrt{1-\eta^2}\right)^2\right\}.$$

The eigen values of M are same as in eq (9). If we put $\alpha = 0$, M converges to $M_{\mu\tau}$ of eq (8). From the eigen vectors of M we can construct the diagonalizing matrix and this can be identified with the PMNS matrix U_{PMNS}.

$$U_{PMNS} = \begin{pmatrix} u_{e1} & u_{e2} & u_{e3} \\ u_{\mu 1} & u_{\mu 2} & u_{\mu 3} \\ u_{\tau 1} & u_{\tau 2} & u_{\tau 3} \end{pmatrix}.$$

Which leads to $tan^2\theta_{12} = |u_{e2}/u_{e1}|^2$, $tan^2\theta_{23} = |u_{\mu 3}/u_{\tau 3}|^2$ and $sin^2\theta_{13} = |u_{e3}|^2$.

Now upon choosing the input $m_0 = 0.0510\ eV$, and the other parameters like $\xi = 0.1733$, $\eta = 0.5655$ and $\alpha = 0.1623$ we obtain,

$$m_1 = 0, m_2 = 0.00884\ eV \text{ and } m_3 = 0.0510\ eV.$$

And hence, the observational mass parameters are calculated as, $\Delta m^2_{21} = 7.815 \times 10^{-5}\ eV^2$, $\Delta m^2_{32} = 2.522 \times 10^{-3}\ eV^2$ and $\Sigma m_i = 0.0598\ eV < 0.28\ eV$ (Cosmological upper bound) [12].

And we obtain the observational parameters related with mixing angles as,

$tan^2\theta_{12} = 0.47$, $tan^2\theta_{23} = 1$ and $sin^2\theta_{13} = 0.026$.

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On changing the parameters η and α to 0.57 and 0 respectively, We obtain the TBM condition i.e., $sin^2\theta_{13} = 0$ and $tan^2\theta_{12} = 0.5$, along-with the observational mass parameters being unaffected.

But in both the cases θ_{23} maintains the maximal condition.

3. Parametrization of the neutrino mass matrix to generate $\theta_{13} \neq 0, \theta_{23} \neq \pi/4$ through a new perturbation matrix M_s'.

TBM mixing is an important mixing scheme which has got a strong theoretical support. This particular mixing can be associated with symmetry group A_4. From phenolomenological point of view, we can investigate on how much perturbation we can provide to the neutrino mass matrix M_s' satisfying TBM condition in terms of a perturbation matrix. And in this process we also try to reduce the number of parameters.

3.1 The texture of M_s'

We first take fix the parameters of m_0, ξ and η in eq(8) in such a way that it can satisfy the TBM mixing condition. Depending upon eq (11), we expect the structure of the perturbating matrix [13, 14] as,

$$M_s' = \begin{pmatrix} r & y & -y \\ y & -x & z \\ -y & z & x' \end{pmatrix}$$

Conducting a numerical scan over M and with $\xi = [0.166,\ 0.176]$, $\eta = [0.5504,\ 0.5790]$ and $\alpha = [0.0773, 0.3146]$ (the ranges of respective parameters are obtained on the basis of interval analysis of the experimental 1σ bounds), and comparing with M_{TBM} , we obtain the texture of M_s' as,

$$M_s' = \begin{pmatrix} 0.2 & 1 & -1 \\ 1 & -0.1 & 0.05 \\ -1 & 0.05 & 0.009 \end{pmatrix} \omega, \qquad (13)$$

Hence for NH model,

$$M_{NH} = \begin{pmatrix} \xi_0\eta_0^2 & -\xi_0\eta_0\sqrt{\frac{1-\eta_0^2}{2}} & -\xi_0\eta_0\sqrt{\frac{1-\eta_0^2}{2}} \\ -\xi_0\eta_0\sqrt{\frac{1-\eta_0^2}{2}} & \frac{1}{2}\{1+\xi_0(1-\eta_0^2)\} & \frac{1}{2}\{\xi_0(1-\eta_0^2)-1\} \\ -\xi_0\eta_0\sqrt{\frac{1-\eta_0^2}{2}} & \frac{1}{2}\{\xi_0(1-\eta_0^2)-1\} & \frac{1}{2}\{1+\xi_0(1-\eta_0^2)\} \end{pmatrix} m_0 + \begin{pmatrix} 0.2 & 1 & -1 \\ 1 & -0.1 & 0.05 \\ -1 & 0.05 & 0.009 \end{pmatrix} \omega \qquad (14)$$

Where ω is a parameter which dictates all the mixing angles and the mass parameters. Depending upon the experimental 1σ range of $tan^2\theta_{12}$ we define a range of ω as $[\omega_1, \omega_2]$ and corresponding to the experimental best-fit value of $tan^2\theta_{12}$ we select a number $\omega = \omega_0$.

3.2 Numerical results

ξ_0 and η_0 are fixed at 0.1733 and 0.5773 respectively and $m_0 = 0.0510$. On plotting $tan^2\theta_{12}$ vs ω, (fig 1) we get $\omega_1 = 0.004$, $\omega_2 = 0.0064$ and $\omega_0 = 0.0053$. The variation of the observational mass parameters and the the mixing angles with respect to the parameter ω are plotted in fig 1. The important results are tabulated below and compared with the best-fit, 1σ, 2σ and 3σ bounds [3].

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Observation al parameters	ω_0	$[\omega_1, \omega_2]$	Best-fit $\pm 1\sigma$	2σ	3σ
$tan^2\theta_{12}$	0.47	[0.43,0.50]	$0.47^{+0.033}_{-0.036}$	0.408-0.538	0.370-0.587
$sin^2\theta_{13}$	0.025	[0.015,0.035]	$0.026^{+0.003}_{-0.004}$	0.019-0.033	0.015-0.036
$tan^2\theta_{23}$	0.919	[0.90, 0.94]	$0.96^{+0.364}_{-0.175}$	0.695 -1.631	0.639-1.778
$\Delta m^2_{21}(10^{-5}eV^2)$	7.58	[7.35, 7.79]	7.62 ± 0.19	7.27-8.01	7.12-8.20
$\Delta m^2_{32}(10^{-3}eV^2)$	2.59	[2.55,2.64]	$2.53^{+0.08}_{-0.10}$	2.34- 2.69	2.26-2.77

4. Summary

We have started with a $\mu-\tau$ symmetric neutrino mass matrix $M_{\mu\tau}(\xi,\eta)$ in such a way that the prediction of θ_{12} (solar angle), depends only upon the parameters η. Keeping other two prediction of mixing angles same as per as the TBM mixing, we can choose η in such a way that it can deviate the prediction of the solar angle from $tan^2\theta_{12}=0.5$. In the next step we introduce extra parameter α in the neutrino mass matrix. The $\mu-\tau$ symmetry is broken an the neutrino mass matrix takes the form of a symmetric matrix $M(\xi,\eta,\alpha)$. On choosing $\alpha=0$, $M \rightarrow M_{\mu\tau}$. M although can give rise to nonzero reactor angle,(θ_{13}) , yet fails to deviate the atmospheric angle (θ_{23}) from maximal condition. In the third step we conduct some numerical scan over all the parameters ξ,η and α, and checked the differences between M and $M_{\mu\tau}$ with $M_{\mu\tau}$ being set at TBM mixing condition. On phenomenological ground we introduce a perturbation matrix M_s with a single parameter ω. This parameter ω not only dictates the mass parameters, solar angle, reactor angle, but also deviates the atmospheric angle θ_{23} from maximal condition. The parametrization we have followed is phenomenological and is not derived from any first principle. We have given attention to the NH case in this piece of work. The generalization of this parametrization to IH [9] and QD [15] cases are in progress. This phenomenological model will be useful in the study of leptogenesis and baryogenesis in estimating the baryon asymmetry of the universe [15].

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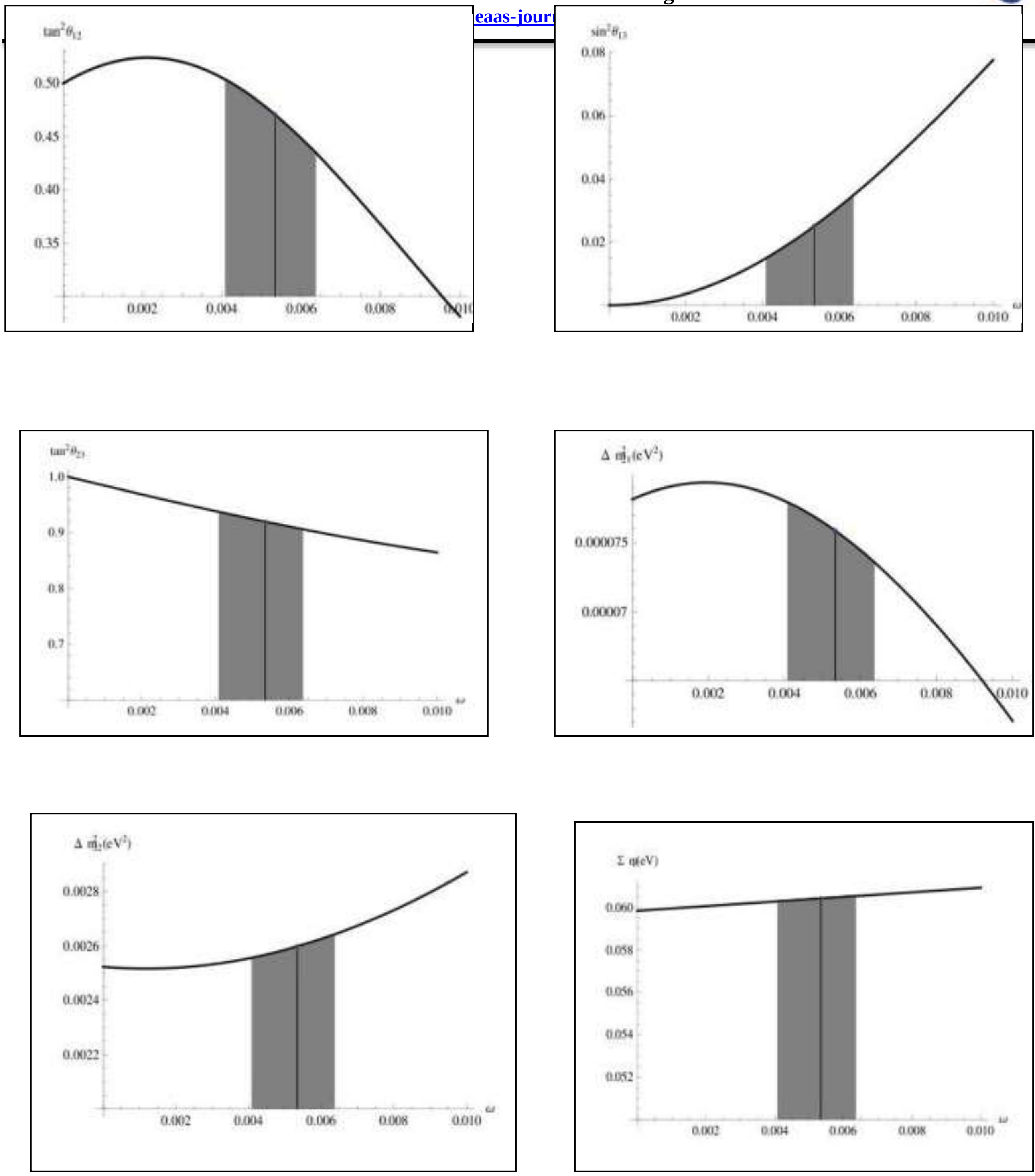

Fig.1 Variation of $tan^2\,\theta_{12}$ with . ω (top-left), variation of $sin^2\,\theta_{13}$ with ω (top-right), variation of $tan^2\,\theta_{23}$ with ω (middle-left), variation of Δm^2_{21} with ω (middle-right), variation of Δm^2_{32} with ω (bottom-left) and variation of Σm_i with ω (bottom-right).

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