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MONTE CARLO ALGORITHM FOR CLASPING SEARCH AND NEUTRON LEAKAGE

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ABSTRACT

A Monte Carlo algorithm has been developed for taking into account neutron leakage effect by a clasping. The algorithm allows one to perform clasping search mode proper value calculations where the clasping is treated as a proper value. For introducing neutron leakage effect, the spatial dependence of neutron instability is come near a single Fourier mode. The particle weight is a complex number.The imaginary part, however, disappears and only the real part needs to be treated if symmetry exists with respect to the direction represented by the clasping and the distributing and splitting neutron emission are isotropous. The algorithm has been verified by comparing buck lings acquire by clasping search mode proper value calculations with those by the diffusion approximation and the B_1 method.When a horizontal calculative field has two orthogonal symmetry planes and the distributing and splitting neutron emission are isotropous the imaginary parts of complex weights disappear. If it does not, complex weights need to be treated. The methodology for treating complex weights has not yet been established and it would be a future work. The newly developed algorithm can also be applied to leakage-corrected Monte Carlo calculations, generating leakage corrected neutron spectra and instability distributions. Although this paper has presented that the algorithm can contribute to the generation of leakage-corrected group continuous is based on Monte Carlo calculation techniques.

Keywords: Monte Carlo algorithm, Clasping, Neutron leakage

INTRODUCTION

Three-dimensional of core calculations by Monte Carlo techniques are gradually becoming a calculation tool for reactor core designs. Meanwhile, continuous energy Monte Carlo calculations are now believed an efficient tool for calculating some neutronic parameters such as diffusion coefficient, which will be used for deterministic core design codes. Yoshioka and Ando [1], and Leppänen [2], have been performed for generating deterministic group continuous by using Monte Carlo techniques. Calculations are usually performed for unit fuel pin cells or fuel assemblies having infinite length in the vertical direction. To obtain accurate group continuous, the effect of neutron leakage to the radial and axial directions has to be taken into account in the Monte Carlo calculations. Neutron leakage corrections are incorporated into deterministic fuel pin cell or fuel assembly calculation codes by using some leakage correction techniques, e.g., the B1 method (Duderstadt and Hamilton, [3]). In this method, the neutron leakage effect is taken into account by specifying the clasping B2 of a reactor core. In the field of Monte Carlo, however, some techniques have been developed to present the neutron leakage effect into fuel pin cell or fuel assembly calculations (Yoshioka and Ando [1]; Fridman and Leppänen [4]; Shim et al. [5]). Leppänen [2], present an additional cross section-like parameter for incorporating the neutron leakage effect. This technique is similar to the calculations of the proper value mode. A term DgB2g is artificially added as simulated absorption where Dg is a diffusion coefficient in the gth energy group and B2g is a geometric clasping. This methodology is based on the assumption that the diffusion holds within reasonable accuracy. A difficulty in estimating the diffusion coefficients for continuous energy Monte Carlo calculations still remains unresolved. The methods are believed unsubstantiated and not recommended to be used for group continuous

generation. Fridman and Leppänen [4] and Shim et al. [5] present a method based on the B1 method for few-group continuous generation. A Monte Carlo calculation is used to produce fine group cross sections used for solving the B1 equations. Then, the B1 equations are solved to obtain the critical neutron spectrum. This method is semi-deterministic, and leakage-corrected effect is not explicitly included in the Monte Carlo calculation. The change of keff caused by neutron leakage is calculated by the perturbation theory. This technique requires the ad joint source distribution that is usually difficult to be estimated by a continuous energy Monte Carlo calculation. Another related work using Monte Carlo technique was performed by Maiorov [6]. To take into account the effect of neutron leakage, particle weights are given by a formula depending on a particle's random walks. Recently, a leakage-corrected Monte Carlo calculation technique was proposed by Yun and Cho [7]. This method presents the effect of neutron leakage by specifying on the outer surfaces of a calculative field instead of introducing a geometric clasping. During the course of a Monte Carlo criticality calculation, an albedo is updated at each generation in such a way that keff is equal to unity. The present paper focuses on determination of the geometric clasping or spatial decompose continuous c in the vertical direction of a fuel pin cell or a fuel assembly. If the geometric clasping in the horizontal direction is larger than the material clasping, the neutron instability decomposes in the vertical direction. The spatial decompose continuous is intimately related to the sub criticality of the fuel pin cell or fuel assembly and can be used as an indicator of nuclear criticality safety. In such a case, a proper value mode neutron transport equation can be defined. Yamamoto and Miyoshi [8] proposed a method to solve the proper value mode equation by Monte Carlo. The method of the proper value mode calculation has been implemented into a continuous energy Monte Carlo code MCNP4C (Briesmeister, [9]). On the other hand, if the geometric clasping in the horizontal direction is smaller than the material clasping, the vertical neutron instability distribution is expressed by a cosine function where Bz is the geometric clasping Where ζ is a direction cosine with the z-axis, and other notations are standard within the nuclear engineering community. This equation is a proper value equation whose proper value is the spatial decompose continuous γ . This equation is to be solved by the repetition method as criticality calculations. To include the last term in Eq. (1) in the z-direction. The vertical clasping Bz is defined by where H is the active height of a core and is the extrapolated length. Some deterministic reactor calculation codes (Fowler et al. [10]) have a function of finding a critical geometric clasping. The present paper is to develop a new algorithm of Monte Carlo calculation method for finding a critical geometric clasping. If certain conditions discussed below are met, the algorithm is also available for fuel pin cell or fuel assembly calculations in which the effect of neutron leakage specified by a clasping is taken into account in the same manner as the B1 method. The B1 method adopts that the leakage current is a function of neutron energy only and that the leakage occurs without directional dependence. On the other hand, the method of this paper adopts that the leakage current depends only on direction. Critical clasping's acquire by the present method are to be compared with the B1 method in one-energy group in a later section.

2. Revisit of spatial decompose continuous search by Monte Carlo method

2.1. Theory of proper value mode calculation by Monte Carlo method

When the horizontal clasping is smaller than the material clasping, spatial decompose continuous search mode calculations by Monte Carlo (Yamamoto and Miyoshi, [8]) are revisited. Suppose that the horizontal clasping in the x and y-directions is larger than the material clasping and that an external neutron source is located in the z-direction. If the subcritical system is homogeneous in the z-direction, the asymptotic neutron instability distribution in the z-direction is given by where is a spatial decompose continuous in the z-direction. The three-dimensional neutron transport equation is rewritten to a two-dimensional form as:

$$\Omega.\nabla\phi(x,y,\Omega,E)+\sum_t(x,y,\Omega,E)\phi(x,y,\Omega,E)=$$
$$=\int d\Omega'\int dE'\phi(x,y,\Omega',E')\sum_S(x,y,\Omega'\to\Omega,E'\to E) \quad (1)$$
$$+\frac{\chi(E)}{4\pi}\int d\Omega'\int dE'\phi(x,y,\Omega',E')\nu\sum_f(x,y,E')+\xi\gamma\phi(x,y,\Omega,E)$$

during the random processes in a Monte Carlo calculation, the differential of the weight of a neutron particle that flies over an infinitesimal distance ds is given by (Yamamoto, [11]):

$$dW=\xi\gamma Wds \quad (2)$$

Thus, after that Si in the ith path with a direction cosine ζ_i , the initial weight Wi changes to:

$$W_{i+1} = W_i \exp(\xi_i \gamma s_i) \quad (3)$$

at this stage γ is an unknown proper value and is given from the previous generation. Unless the weight changes during its path, the product of the track length length S_i and its weight W_i is given by W_iS_i. The weight, however, changes during its path. After the *i*th path with the distance S_i, the, the product of the track length and the particle's weight is given by:

$$TL_i = \int_0^{s_i} W_i \exp(\xi_i \gamma s_i') ds_i' = W_i \frac{\exp(\xi_i \gamma s_{i)} - 1}{\xi_i \gamma} \quad (4)$$

TL_i in Eq. (4) is the track length estimator of neutron instability when neutron weight is a continuously changing function across the path. This is not only to the calculation of k_{eff} but to all tallies as well. Major modifications to a Monte Carlo criticality calculation code needed for γ proper value mode calculations are implementation of Eqs. (3) and (4). All other procedures for criticality calculations can be utilized for γ proper value mode calculations without any modifications. After all random processes within one generation are completed, the γ' proper value is estimated as:

$$\gamma' = \frac{N.W_S + \sum_i \Delta W_i - \sum_i \nu \Sigma_f .TL_i}{\sum_i TL_i .\xi_i} \quad (5)$$

where *i* is summarize over all paths within one generation, N is the number of source per generation, $\mathrm{D}W_i = W_i(\exp(z_i g s) - 1)$ and W_s is the weight of particles that start from splitting source sites. The starting weight W_s is given by M/N where M is a nominal source size. N points come from the splitting source points of the previous generation. Thus, the total source weight per generation is a continuous M. An indicator of, k, is defined as:

$$k = \frac{\sum_i \nu \Sigma_f .TL_i}{\mathrm{M}} \quad (6)$$

This indicator has the same definition as k_{eff} and is supposed to be unity if converged. The γ is used for the next generation calculation is determined in such a way that k approaches unity as:

$$\gamma_{j+1} = \gamma_j + c(1 - k_j) \quad (7)$$

where the *j* stands for the generation number and γ is an unreasonable positive value. The γ are averaged over all generations after generations to determine a final γ. The γ' mode calculation is extends to infinity length along the z-axis.

2.2. Confirmation of mode calculation by Monte Carlo method

The algorithm of the γ mode calculation by Monte Carlo method is installed into the continuous energy Monte Carlo code MCNP4C. The γ mode calculation was numerically verified by comparing decompose continuous calculated by the γ mode calculations with those by fixed source calculations for uranyl nitrate aqueous fuel solutions (Yamamoto and Miyoshi, [8]). In the fixed source calculations, neutron instability distributions in the z-direction were calculated and the decompose continuous were acquire by fitting the instability distributions. This paper presents a numerical confirmation using a homogeneous, one-energy group problem with isotropous distributing in the laboratory system since in this situation an accurate reference solution can be acquire by the diffusion theory. A test Monte Carlo code has been prepared for the numerical test. Suppose that a rectangular parallelepiped with the side length of 42.123 cm is composed of a homogeneous multiplying medium. The one-group continuous is shown in Table 1. The spatial decompose continuous based on the diffusion theory is given by:

$$\gamma_1 = \sqrt{B_X^2 + B_y^2 - B_m^2} \quad (8)$$

where,

$B_X^2 = B_y^2 = (\pi / (H + 2\lambda))^2$, (H) is side length ,

$B_m^2 = (\nu \sum_f - \sum_a) / D$,

$D = 1/(3\sum_t)$, $\lambda = 0.7104/\sum_t$

The spatial decay constant γ_1 is based on the diffusion in the horizontal direction as well as in the vertical direction. To improve the accuracy of is as the reference solution, another spatial decompose continuous is defined by:

$$\gamma_2 = \sqrt{B_h^2 - B_m^2} \quad (9)$$

where $B_h^2 = (\nu\Sigma_f / k_{in} - \Sigma_a)/D$, K_{in} is neutron multiplication factor of the rectangular parallelepiped with infinite length in the vertical direction. K_{in} was calculated by a Monte Carlo criticality calculation. Thus, the spatial decompose continuous γ_2 is more accurate than γ_1 since the horizontal direction is treated by the neutron transport theory. Another spatial decompose continuous γ_3 is acquire by instability distribution in the vertical direction to $\exp(\gamma_3 z)$. The instability distribution was calculated by a fixed source calculation for the rectangular parallelepiped with infinite length in the vertical direction. In the calculation for the instability distribution, an external neutron source was placed at z = 0. The calculated vertical instability distribution is shown in Fig.1. The instability is fitted in the region far enough away from the source to exclude the effect of higher harmonics.

Table 1- Group constants and calculation results for exponential flux distribution

Parameters	Values
∑,t (cm^{-1})	0.822222
∑ *a*,(cm^{-1})	0.1192222
∑,f (cm^{-1})	0.0402222
γ	3
k_{in}	0.973798 ± 0.000003
γ_1 (cm^{-1})[a]	0.0903878
γ_2 (cm^{-1})[b]	0.090102 ± 0.000012
γ_3 (cm^{-1})[c]	0.090345 ± 0.000002
γ_e (cm^{-1})[d]	0.090489 ± 0.000025
k	0.9992222 ± 0.000006

a) Spatial decay constant defined by Eq. (8).

b) Spatial decay constant defined by Eq. (9).

c) Spatial decay constant fitted to exponential function.

d) Spatial decay constant by c-eigenvalue mode calculation.

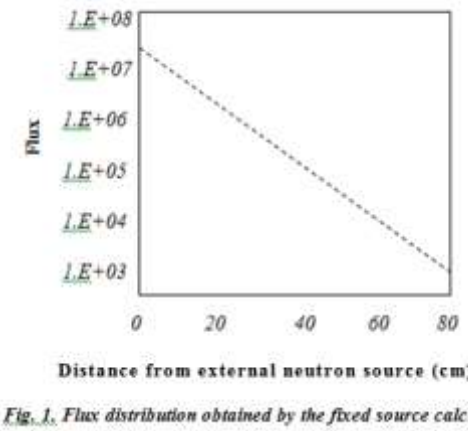

Fig. 1. Flux distribution obtained by the fixed source calculation.

The γ_e was acquire by the γ mode calculation with 55,000 neutrons per generation, skipping 250 generations and running 85,000 active generations. At the same time, the indicator k defined by Eq. (6) was calculated. The calculation results of the spatial decompose continuous are shown in Table 1. Four γ values in Table 1 relatively agree with each other. γ_1 and γ_2 slightly differ from γ_3 and γ_e due to the inaccuracy involved in using the diffusion approximation. The γ_e shows excellent agreement with the decompose continuous γ_3 that is rigorously based on the neutron transport theory. Therefore, one may conclude that the algorithm shown in Section 2.1 provides an accurate γ Eq. (1) and the algorithm is verified numerically.

3. Clasping search mode calculation by Monte Carlo method

3.1. Theory of clasping search mode calculation

This section deals with cases where the horizontal clasping is smaller than the material clasping. According to that the energy-dependent horizontal instability and the energy-independent vertical instability are separable and the vertical instability distribution is characterized by a clasping mode as:

$$f(x,y,z,W,E) = f(x,y,W,E)\exp(iBz) \quad (10)$$

where *i* is the imaginary unit, B is geometric clasping in the z-direction. Here, one of the spatial dependence in the z-direction by a single Fourier mode, $\exp(iBz)$. It is assumed that the vertical instability is independent of neutron energy. This assumption is not correct but is forced to be presented for representing the leakage effect by the single factor B as in the widely-used B_1 method. Substitute Eq. (10) into the three-dimensional neutron transport equation. Then, after a algebraic manipulation, one obtains a two-dimensional transport equation including the vertical clasping B:

$$\Omega.\nabla\phi(x,y,\Omega,E) + \sum_t(x,y,E)\phi(x,y,\Omega,E) =$$
$$= \int d\Omega' \int dE' \phi(x,y,\Omega',E') \sum_s(x,y,\Omega' \to \Omega, E' \to E) \quad (11)$$
$$+ \frac{\chi(E)}{4\pi} \int d\Omega' \int dE' \phi(x,y,\Omega',E') \nu \sum_f(x,y,E') - iB\xi\phi(x,y,\Omega,E)$$

Where ζ is a direction cosine with the *z*-axis. Eq. (11) is very similar to Eq. (1) .To incorporate the last term in Eq. (11) into random processes of a Monte Carlo calculation, we follow the same procedure as in Section 2.1. The differential of the weight of a neutron particle caused by flying over infinitesimal distance ds is given by:

$$dW = -iB\xi W ds \quad (12)$$

Thus, after a particle flies over a distance S_j in the *j*th path with a direction cosine ζ_j, the initial weight W_j changes to:

$$W_{j+1} = W_j \exp(-iB\xi_j S_j) =$$
$$= W_j(\cos(B\xi_j S_j) - i\sin(B\xi_j S_j)) \quad (13)$$

The product of the track length and the particle's weight after the *jth* path with a distance S_j is given by:

$$TL_j = \int_0^{S_j} W_j \exp(-iB\xi_j S_j') dS_j =$$
$$= W_j \frac{\exp(-iB\xi_j S_j) - 1}{-iB\xi_j} = \quad (14)$$
$$= W_j \left(\frac{\sin(B\xi_j S_j)}{B\xi_j} + i \frac{\cos(B\xi_j S_j) - 1}{B\xi_j} \right)$$

While the initial weight of a particle starting from a splitting source is a real number, the subsequent weights can be complex numbers. If the splitting neutron emission is isotropous in the laboratory system, the imaginary part of W_{j+1} in Eq. (13), $W_j \sin(iBzs_j)$, appears with the same frequency as $W_j \sin(iBzs_j)$. Furthermore, if the distributing is isotropous, the imaginary parts of weights disappear for the same reason. Since the calculative field is homogeneous in the z-direction, the weights are symmetric with respect to the z-direction. In the same way, the imaginary part of TL_j in Eq. (14), $(\cos(Bx_j S_j) - 1)/B x_j$, which is an odd function of x_j, is supposed to disappear. Therefore by imaginary parts in Eqs. (13) and (14) can be omitted only if the splitting neutron emission and distributing are isotropous. Then, under such conditions, Eqs. (13) and (14) are rewritten as follows:

$$W_{j+1} = W_j \cos(B\xi_j S_j) \quad (15)$$

$$TL_j = W_j \frac{\sin(B\xi_j S_j)}{B\xi_j} \quad (16)$$

W_j and TL_j can be negative when $|Bzs_j| > p/2$. When B is large (i.e., large neutron leakage in the z-direction) and a mean free path is long, a negative weight tends to occur in Eq. (15). Even after a weight turns negative, the particle with a negative weight keeps being followed. The negative particle weight is treated the same way as if it were positive.

Discarding the negative weights is equivalent to shortening the path length that $|Bzs_j| < p/2$. The proposed method, however, adopts an infinite dimension in the z-direction. The weight given by Eq. (15) is a periodic function of the path length, and the negative weight becomes positive again. If the path length is very long in such a way that $|Bzs_j| > 3p/2$, the weight may become larger than what it was before the path started. This problem may be emphasized in reactor systems with gas-filled regions. Such heterogeneous systems including void regions are problematic and should be treated as a future work. If a particle collides, (n ≥ 1) splitting source sites are stored for use as splitting sources in the next generation where:

$$n = Int(|W|) \frac{n\Sigma_f}{\Sigma_t} + R \quad (17)$$

Int(...) is the integer part, *R* is uniform simulated random number in [0, 1). If W < 0, a negative weight

is a signed to the *n* splitting sources. After all random processes within one generation are completed, the proper value B' is estimated as:

$$B' = \frac{N W_S + \sum_j \Delta W_j - \sum_j n \sum_f TL_j}{\sum_j A_j} \quad (18)$$

Where *j* is summarize over all paths within one generation, *N* is the number of source per generation, $\Delta W_j = W_j(\cos(Bz_j S_j) - 1)$, and:

$$A_j = \mathrm{Re}\left[ix_j \int_0^{S_j} W_j \exp(-iBx_j S') ds' \right] = \frac{W_j}{B}\left(\cos(Bx_j S_j) - 1\right) \quad (19)$$

Here, Re[...] is a real part, and $N = N_p - N_m$ where N_p and N_m are the numbers of splitting source sites with positive and negative weights, respectively. W_s is given by $W_s = M/N$ where M is a nominal source size. k_{eff} of Eq. (11) is supposed to be unity and is given by:

$$k_{eff} = \frac{\sum_j n \sum_f .TL_j}{M} \quad (20)$$

The clasping *B* used for the next generation calculation is determined that k_{eff} approaches unity as:

$$B_{l+1} = B_l + d(K_{eff}^l - 1) \quad (21)$$

Where *l* stands for the generation number and *d* is an unreasonable positive value. Splitting sources with negative weights must be cancelled at the end of each generation. This present paper uses the ‘‘procedure’’ for the weight cancellation (Yamamoto, [12]). A whole region is divided into a large number of small regions. Splitting sources with positive and negative weights are accumulated in the bins. The number of splitting source particles in the *jth* bin is given by $n_j = n_{jp} - n_{jm}$ where n_{jp} and n_{jm} are the number of splitting sources with positive and negative weights, respectively. n_j splitting sources that are used for the next generation’s starters are distributed uniformly within the *jth* bin.

3.2. Confirmation of clasping search mode calculation

As shown in Section 3.1, homogeneous one-energy group problems were used for numerical confirmation of the clasping search mode calculation. The one-energy group does not limit the generality of this developed method. It is straightforward to extend this method to general continuous energy problems. First, one-energy group problems that have infinite dimension in the horizontal direction were believed. In this case, the neutron instability in the neutron transport equation, Eq. (11), becomes a function of direction Ω only as follows:

$$\sum_t f(\Omega) - \frac{1}{4p}(\sum_s + n\sum_f) \int_{4p} f(\Omega')d\Omega' + iBxf(\Omega) = 0 \quad (22)$$

This problem is suitable for confirmation of the clasping search mode calculation model since no neutron transport in the horizontal direction is presented. The proper values B’s of Eq. (22) were acquire by search mode calculations with 55,000 neutrons per generation, skipping 250 generations and running 80,000 active generations. Since this problem adopts that the splitting neutron emission and distributing are isotropous, the imaginary parts of weight cancel out and the weights are expressed by using Eqs. (15) and (16). The calculations were performed for several cases. The group continuous and calculation results are shown in Table 2. B_0 is the clasping based on the diffusion and is given by $B_1 = \sqrt{B_m^2}$, B_e in Table 2 is the clasping acquire by the clasping search calculation. In cases 1, 2, and 3 in Table 2, we have a negative weight in each path is less than 3×10^{-7} even when n = 1 or -1 Thus, the effect of negative weights is almost negligible. In case 4, the probability is 3% when n = 1 or -1 and the fraction of splitting sources with negative weights is 1.3%. Thus, the effect of negative weights is significant in case 4. In cases 1, 2, and 3, the results of the clasping search calculations agree with those by the diffusion within 0.3%. On the other hand, in case 4 a notable difference between B_0 and B_e is found. Fig.2 shows the dependences of neutron instability for cases 2 and 4 that were acquire during the clasping search mode calculations. The dependence for case 2 is almost flat and, the slight dependence can be accurately come near by a linear function of the direction cosine ζ, which is one of the requisites of the diffusion approximation. This fact means that the diffusion gives a quite accurate clasping. For case 4, however, the dependence clearly

deviates from a linear function of the direction cosine ζ. Thus, B_0 in case 4 is believed to be biased, and the difference between B_0 and B_e may be due to the inaccuracy of B_0. To obtain a more accurate clasping, an instability distribution in the z-direction was calculated for a horizontally infinite but vertically finite slab that is nearly critical (k_{eff} = 1.000008 ± 0.000003). The thickness of the critical slab was 15.7521 cm. The vertical instability distribution was fitted to a cosine function, the vertical clasping. The vertical instability distribution, however, is not exactly cosine shape due to neutron transport transient effects near the outer boundaries. The instability distribution only near the center of the slab was used for the fitting. The instability distribution and fitted cosine curve are shown in Fig. 3. The area used for the fitting was 0 –1.35 cm. The clasping acquire from the instability distribution, B_c, is given in Table 2. The clasping B_c agrees well with B_e, which may show confirmation of the clasping search mode calculation with large neutron leakage. Another confirmation can be done by comparing with the B_1 method. The B_1 equations are given in (Duderstadt and Hamilton, [3]) as:

$$iBJ(u) + \Sigma_t(u) f(u) = \int_0^u du' \Sigma_{s0}(u' \to u) f(u') + \frac{1}{k_{eff}} c(u) \int_0^{\infty} du' \nu \Sigma_f(u') f(u') \tag{23}$$

$$\frac{iB}{3} f(u) + g(B,u) \Sigma_t(u) J(u) = \int_0^u du' \Sigma_{s1}(u' \to u) J(u') \tag{24}$$

$$g(B,u) = \frac{\left(\frac{B}{\Sigma_t(u)}\right)^2 \tan^{-1}\left(\frac{B}{\Sigma_t(u)}\right)}{3\left(\frac{B}{\Sigma_t(u)}\right) - \tan^{-1}\left(\frac{B}{\Sigma_t(u)}\right)} \tag{25}$$

Table2-Group constants and calculation results for the buckling search mode calculations with infinite dimension in the horizontal direction.

	Case 1	Case 2	Case 3	Case 4
$\sum t$ (cm^{-1})	1.0	0.822222	1.666556	0.322222
$\sum a$ (cm^{-1})	0.2	0.12	0.119333	0.05
$\sum f$ (cm^{-1})	0.0657	0.040	0.040745	0.024659
γ	3	3	3	3
B_0(cm^{-1})[a]	0.03458	0.08559	0.11178	0.15487
B_e(cm^{-1})[b]	0.03368 ± 0.00006	0.08665 ± 0.00001	0.11174 ± 0.00002	0.15922e ± 0.00001
B_c (cm^{-1})[c]	-	-	-	0.15987 ± 0.00005
B_1(cm^{-1})[d]	0.03398	0.08648	0.11165	0.15857
k_{eff}	1.000000 ± 0.000003	1.000000 ± 0.000003	1.000000 ± 0.000003	1.000000 ± 0.000005

a) Buckling by diffusion approximation.

b) Buckling by buckling search mode calculation.

c) Buckling by fitting to cosine function.

d) Buckling by the B1 method.

e) If the negative weights are discarded, Bc = 0.163178 ± 0.00001 cm-1.

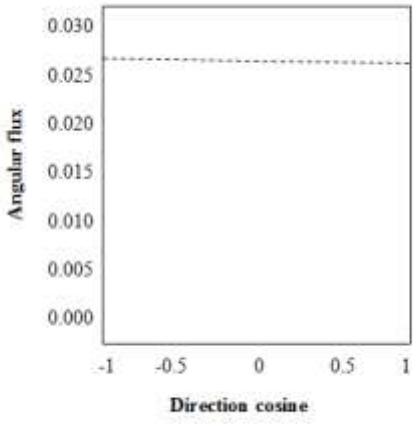

Fig. 2. Angular dependence of neutron flux for cases 2 and 4.

Where u is the neutron lethargy, J(u) is neutron current, $\sum_{so}$ is isotropous component of the distributing cross section, and $\sum_{s1}$ is the linearly an isotropous distributing component of the distributing cross section. The equations are modified from the original ones in the literature. Assuming that the distributing is isotropous (i.e,. $\sum_{s1} = 0$) and the neutron is monoenergetic, Eqs. (23) and (24) are simplified to:

$$(\text{å}_t - \text{å}_S)g(\mathrm{B}) + \frac{\mathrm{B}^2}{3} = \frac{1}{k_{eff}} n\,\text{å}_f\, g(\mathrm{B})\,\text{å}_t \qquad (26)$$

where the variable u and the subscripts for the distributing orders are omitted. Using Eq.(26), the critical buck lings for $k_{eff} = 1$ can be easily acquire by iterative calculations. The critical buck lings acquire by Eq.(26) are listed in Table 2. The critical bucks lings acquire by the B_1 method agree very well with the Monte Carlo calculations for all cases. If the negative eights are immediately discarded, the calculated critical clasping is $0.163188 \pm 0.00001\, cm^{-1}$ which is definitely wrong. The proposed algorithm for the clasping search calculation by Monte Carlo method can reproduce very accurately the B_1 method. For numerical confirmation of the method for solving Eq. (11), a rectangular parallelepiped with a finite side length in the horizontal direction was believed for each of cases 1–4. The group continuous, dimensions, and calculation results are shown in Table 3 .B_0 in Table 3 is a vertical clasping and is defined by:

$$B_0 = \sqrt{B_m^2 - B_x^2 - B_y^2} \qquad (27)$$

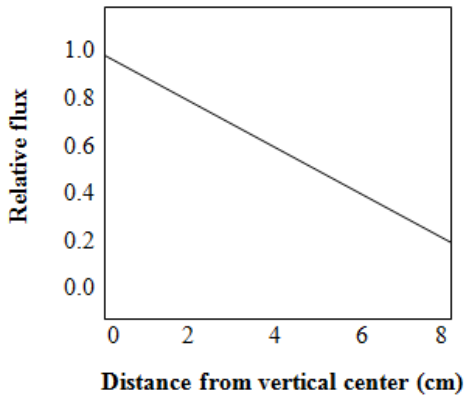

Fig. 3. Vertical flux distribution for case 4

Another vertical clasping B2 is given in Table 3, which is defined by:

$$B_2 = \sqrt{B_m^2 - B_h^2} \qquad (28)$$

where $B_h^2 = (n\,\text{å}_f / k_{in} - \text{å}_a)/D$, $k_{im} = k_{eff}$ of the rectangular parallelepiped with infinite length in the vertical direction. k_{in} was calculated by a Monte Carlo criticality calculation. B_2 is more accurate since the horizontal direction is treated by the neutron transport theory. Excellent agreement is found between B_2 and B_e except for case 4f where anisotropy of neutron instability is significant due to the large vertical clasping and long mean free path.

4 .Leakage-corrected calculation by Monte Carlo method

Suppose that a clasping in the vertical direction is known a priori and the horizontal direction is treated, including the horizontal neutron leakage, by the neutron transport theory. The neutron transport equation for this case is given by:

$$\begin{aligned} &\text{W.Ñ}f(x,y,\text{W},E) + \text{å}_t(x,y,E)f(x,y,\text{W},E) \\ &= \text{ò}\,d\text{W}'\text{ò}\,dE'f(x,y,\text{W}',E')\text{å}_S(x,y,\text{W}'\text{®}\,\text{W},E'\text{®}\,E) \qquad (29) \\ &+ \frac{1}{k_{eff}}\frac{c(E)}{4p}\text{ò}\,d\text{W}'\text{ò}\,dE'f(x,y,\text{W}',E')n\,\text{å}_f(x,y,E') \\ &- i\mathrm{B}xf(x,y,\text{W},E) \end{aligned}$$

Where B is not an proper value but a user-specified clasping in the z-direction. The calculation algorithm

for solving Eq.(29) is almost the same as conventional Monte Carlo criticality calculations for seeking k_{eff}. The exceptions are that a particle weight and the product of a track length and particle weight are given by Eqs. (15) and (16) respectively. k_{eff} appears explicitly as an proper value of Eq.(29).

The clasping B is not updated and is fixed at the user-specified value throughout the calculation. The calculation in which neutron leakage in the vertical direction is corrected by a user-specified vertical clasping is easier than the clasping search mode calculation since the clasping need not be updated. By solving Eq. (29), one can obtain a leakage-corrected neutron instability distribution and neutron spectrum, which will subsequently be used for generating group continuous. Next, consider a constituent unit of a whole core configuration such as a unit fuel, a single fuel assembly, or multi-fuel assemblies. The reflective conditions are require on the boundary surfaces of the constituent unit. Although the neutron transport phenomenon is treated by Monte Carlo method within the range of the unit, the effects of vertical and horizontal neutron leakage are presented by vertical and horizontal buck lings, respectively. Again, the unit is homogeneous and infinite in the z-direction. Assume that the three-dimensional instability distribution in an x–y–z Cartesian coordinate system is characterized by a simple clasping mode as:

$$f(x,y,z,W,E) = f(x,y,W,E)\exp(i(B_x x + B_y y + B_z z)) \quad (30)$$

where B_x, B_y and B_z are geometric direction buck lings in x, y, and z-directions, respectively. Substituting Eq. (30) into the three dimensional neutron transport equation, one obtains a two-dimensional transport equation that is similar to Eq. (11). However, the last term is different and is given by:

$$-i(B_x m + B_y h + B_z x) f(x,y,W,E) \quad (31)$$

where *u*, *n* are direction cosines with the x- and y-axes, respectively. Eqs. (13) and (14) are, respectively, rewritten as:

$$W_{j+1} = W_j \exp(-i(B_x m_j + B_y h_j + B_z x_j)S_j) = W_j(\cos((B_x m_j + B_y h_j + B_z x_j)S_j) - i\sin((B_x m_j + B_y h_j + B_z x_j)S_j)) \quad (32)$$

$$TL_j = \int_0^{S_j} W_j \exp(-i(B_x m_j + B_y h_j + B_z x_j)S)dS = W_j\left[\frac{\sin((B_x m_j + B_y h_j + B_z x_j)S_j)}{B_x m_j + B_y h_j + B_z x_j} + i\frac{\cos((B_x m_j + B_y h_j + B_z x_j)S_j) - 1}{B_x m_j + B_y h_j + B_z x_j}\right] \quad (33)$$

Consider that a horizontal calculative field has two orthogonal symmetry planes and the distributing and splitting neutron emission are isotropous. For such a case, Gelbard and Lell [13] mentioned‘‘ a clasping cannot exist anywhere in the lattice one cannot fit a cosine very well to any segment of this curve’’. However, suppose that one continues to presume, roughly, a cosine shape even for an asymmetric lattice cell and the horizontal buckling B_x and B_y can be defined within a certain range of accuracy. Then, one option is to neglect the imaginary parts of Eqs. (32) and (33)which however may present errors that are difficult to be quantitatively estimated. Another option is to treat complex weights defined by Eq. (32). A deterministic core calculation code equipped with the B_1 method could be used for confirmation of the new algorithm that handles complex weights. When one is interested in a critical clasping search calculation in a three-dimensional configuration, three direction buckling, B_x, B_y and B_z, need to be determined. However, the combination of three direction buckling is unreasonable. Thus, some restraint conditions are required on the combination of B_x, B_y and B_z. For instance, B_x and B_y are fixed at continuous and then only B_z is searched. Or, B_x is equal to B_y and the ratio of the horizontal clasping to the vertical clasping is fixed and so on.

5. CONCLUSIONS

This paper has used a Monte Carlo calculation algorithm for clasping search mode calculations where the horizontal direction is treated by neutron transport calculation and neutron leakage effect in the remaining vertical direction which is taken into account by the vertical clasping. When the horizontal clasping is larger than the material clasping, the

vertical neutron instability distribution takes the form of an exponential function. The spatial decompose continuous of the vertical neutron instability distribution can be acquire as a proper value by solving a clasping search mode proper value equation. The proper value can be verified by comparing with the spatial decompose continuous calculated by a fixed source problem. On the other hand, when the horizontal clasping is smaller than the material clasping, a proper value equation is a clasping in the vertical direction can be defined by the vertical direction by a single Fourier mode, exp(iBz). The proper value equation includes a complex term, which produces complex particle weights during the Monte Carlo random processes. However, if the calculative field is uniform in the vertical direction and the distributing and splitting neutron emission are isotropous, the weights turn out to be symmetric with respect to the vertical direction, which eliminates the imaginary parts from the complex weights. Thus, only the real parts remain in the random processes. When a clasping is large and a neutron's mean free path is long, a negative weight may appear during the random processes. The negative weight is treated the same way as the positive weight. When appearance of negative weights is significant, the diffusion becomes worse due to notable anisotropy of neutron instability. In such a case, confirmation by comparing with the diffusion is inaccurate. The clasping acquire by the clasping search mode proper value calculation was compared with the one acquire by the instability distribution or by the B1 method. Both buckling agree well, and the clasping search mode calculation has been verified for large neutron leakage systems. This Monte Carlo algorithm for introducing neutron leakage effect by a clasping can also be applied to leakage-corrected Monte Carlo calculations. The buckling in the x, y, and z-directions are fixed at user-specified values, and then Keff, neutron spectrum, splitting source distribution and so on are calculated for generating group continuous used for a subsequent reactor core design. When a horizontal calculative field has two orthogonal symmetry planes and the distributing and splitting neutron emission are isotropous the imaginary parts of complex weights disappear. If it does not, complex weights need to be treated. The methodology for treating complex weights has not yet been established and it would be a future work. The newly developed algorithm for solving the clasping search mode of proper value equation was verified through comparison with the clasping does acquire by the diffusion and the B1 method. As long as the diffusion is reasonable, both clasping's acquire by the clasping search mode of proper value and the diffusion agree well, which exhibits confirmation of the new algorithm.

REFERENCES

1. Yoshioka, K., Ando, Y., 2010. Multigroup scattering matrix generation method using weight-to-flux ratio based on a continuous energy Monte Carlo technique. Journal of Nuclear Science and Technology 47, 908–916.
2. Leppänen, J., 2007. Development of a New Monte Carlo Reactor Physics Code. VTT Publications 640.
3. Duderstadt, J.J., Hamilton, L.J., 1976. Nuclear Reactor Analysis. John Wiley & Sons, New York.
4. Fridman, E., Leppänen, J., 2011. On the use of the Serpent Monte Carlo code for few group cross section generation. Annals of Nuclear Energy 38, 1399–1405.
5. Shim, H.J., Han, B.S., Jung, J.S., Park, H.J., Kim, C.H., 2012. McCARD: Monte Carlo code for advanced reactor design and analysis. Nuclear Engineering and Technology 44, 161–176.
6. Maiorov, L.V., 1985. Calculating neutron-flux functionals by the Monte Carlo method in breeder systems with leakage specified by a geometric parameter. Translated from Atomnaya Énergiya 58, 93–96.
7. Yun, S., Cho, N.Z., 2010. Monte Carlo depletion under leakage-corrected critical spectrum via albedo search. Nuclear Engineering and Technology 42, 271–278
8. Yamamoto, T., Miyoshi, Y., 2003. An algorithm of α and c-mode eigenvalue calculations by Monte Carlo method. In: Proc. 7th Int. Conf. on Nuclear
9. Briesmeister, J.F., (Ed.) 2000. MCNP – A General Monte Carlo N-particle transport code, Version 4C. LA-13709-M.
10. Fowler, T.B., Vondy, D.R., Cunningham, G.W., 1969. Nuclear Reactor Core Analysis Code: CITATION, ORNL-TM-2496, Rev. 2.
11. Yamamoto, T., 2011a. Higher order a mode eigenvalue calculation by Monte Carlo power iteration. Progress in Nuclear Science and Technology 2, 826–835.

12. Yamamoto, T., 2011b. Non-region wise weight cancellation for Monte Carlo higher order criticality calculations using kernel density estimator. Annals of Nuclear Energy 38, 2515–2520.
13. Gelbard, E.M., Lell, R., 1977. Monte Carlo treatment of fundamental-mode neutron leakage in the presence of voids. Nuclear Science and Engineering 63, 9–23.