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SIMULTANEOUS SCHEDULE DESIGN & ROUTING WITH MAINTENANCE CONSTRAINTS FOR SINGLE FLEET AIRLINES

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ABSTRACT

We propose to find simultaneously the flight schedule design and the aircraft routing, considering at the same time maintenance requirements. This approach leads to a closer global optimal solution than the typical sequential one, and it is practical for the case of airlines with a single fleet, such as many emerging young low-cost airlines.

The formulation of this optimization problem is a Longest Path Problem with maintenance requirements as side constraints. It is based on a network diagram where a flight schedule is defined as the set of legs included in all routes to be flown. Then an algorithm to solve this integrated problem is proposed. It is a greedy heuristic that starting from a given large number of potential flight legs, it identifies at each cycle the route that provides the maximum profit.

The model was tested using 7 instances extracted from real-world timetables of three Latin American airlines. It was found that the model can be solved using standard type of computers in less than a minute for single fleet airlines with less than 10 aircrafts.

Results were compared to the ones obtained from a scheduling model that does not include maintenance constraints. This approach allowed comparing the results with lower bounds of the cost per flight times. It was found that for most of the instances, the proposed approach generates the same values.

Keywords: flight schedule design, maintenance and routing, single fleet airlines

1 INTRODUCTION

For major airlines, the unmanageable size and complexity of the planning process has resulted in the decomposition of the overall problem into a set of sub problems that are solved sequentially.

Optimization models generated for these large companies usually are of huge sizes since the problems are usually combinatorial and these companies have dozens of fleets (aircrafts with the same characteristics) and hundreds of destinations. This fact has forced to consider the planning process in sequential fashion as shown in Fig. 1 (Klabjan, et al., 2002), (Sandhu & Klabjan, 2007). Currently there exists several highly costly software that using this strategy are helping these large companies to elaborate their planning process.

However this highly costly software and their sequential strategy is not necessarily the most appropriate for small and/or single fleet airlines that are becoming to be well known as low-cost airlines. Therefore two or more

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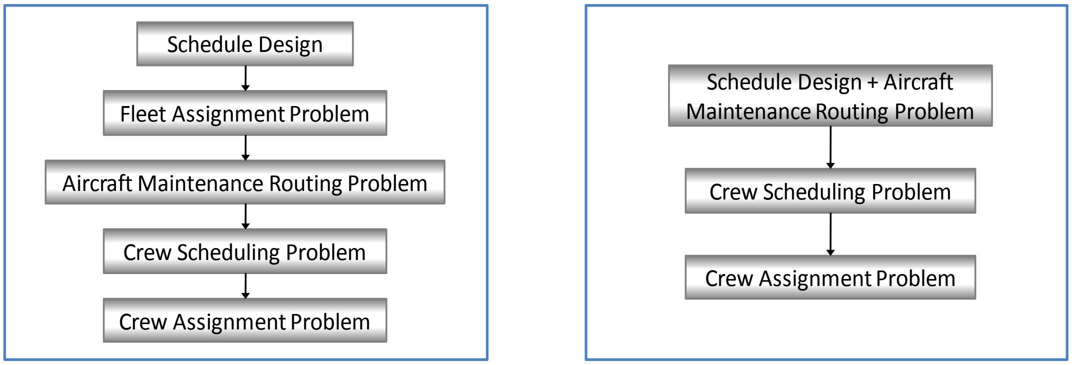
problems of the planning process could be solved simultaneously, yielding solutions potentially better for their specific needs.



(a) (b)

Fig. 1 *(a) Traditional sequential planning process for large airline companies,*
(b) partially-simultaneous planning process proposed for single fleet airline companies

Currently, a boom of new low-cost airlines is taken place. 70 out of the 109 airlines listed in one of the main airlines engine search on internet are classified as low-cost (flylowcostairlines, 2011), 25 of them were created in the last decade. They offer continental or domestic service mainly, have strong hub structure networks with one or two hubs, and have an average fleet size of 12 aircrafts with a third part having 5 or less. From this group, 48% has a single fleet and 32% has 2 fleets. This reference includes airlines operating in all the continents, except from Center and South America. However, it is known that many of the airlines that operate in this part of the American continent share the characteristics mentioned above.

Clearly, for all these small airlines it is not required to solve the fleet assignment problem. Either, they have only one fleet or they already have pre-assigned their few fleets to sets of destinations considering constraints such as aircraft size or travel distance, especially in the case when they have strong hub networks.

For these single fleet airline companies, the sizes of the optimization problem for the planning process are reduced drastically. Therefore the sequential approach is not necessary any longer and two or more problems of the planning process could be solved simultaneously. Solutions obtained in this manner are potentially better than the ones that it can be obtained using the models created for the large airline companies.

This work focuses on finding aircraft routes and the corresponding flight schedule, considering maintenance constraints that maximize the profit function for the case of single fleet airlines companies.

The major contributions of this paper are: a) a methodology to define flight schedule from the routing solution b) the inclusion of side constraints in the routing problem to easily consider maintenance opportunities, and c) a proof of concept of the proposed approach by comparing results with lower bounds for the case of seven instances from 3 different Latin American airlines.

1.1 Flight Schedule Design Problem (FSDP)

Flight Schedule Design Problem (FSDP) is the problem of defining the flight legs and their departure times to be offered by one airline. This information addresses issues related to the network structure, destinations, frequency, etc., It is the most important product of an airline and defines the airline's competitive position in the market. (Barnhart & Cohn, 2004).

Designing a flight schedule is a very complex process because of the inherent size of the problem, difficulties in developing a feasible network, the availability and accuracy of data, and the strong relationship between market

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demand, the airline's schedule profitability, and competitors' service. (Kang, et al., 2007). For these reasons, this process is usually done manually, with limited optimization.

Reviews of network design models are done in (Minoux, 1989) and specifically for the airline industry in (Barnhart, et al., 2003). A common approach uses incremental optimization. This method starts with a base schedule and additions or deletions are made looking for improvements of the objective function. Authors point out computational difficulties of starting from scratch, steadiness of network structures and fairly degree of consistency (Lohatepanont & Barnhart, 2004).

1.2 Aircraft Maintenance Routing Problem (AMRP)

Aircraft Maintenance Routing Problem (AMRP) is the problem of assigning, at a minimal cost, routes of flight legs (or just legs) to be flown by each individual aircraft from a single fleet in such a way that each leg is covered exactly once while ensuring appropriate aircraft maintenance (Cordeau, et al., 2001). This problem is NP-Hard, since it is a special case of the general Vehicle Routing Problem (VRP) with side maintenance constraints.

A review of both exact and heuristic methods to solve VRPs concerning the distribution of goods between depots and final users is presented in (Toth & Vigo, 2002). They report several solution algorithms including linear and Lagrangean relaxations, constructive and two-phase heuristics, and metaheuristics. The last ones have shown to be fast for large practical applications.

AMRP is usually solved for a given and fixed flight schedule that is typically designed manually or with limited optimization such as incremental improvements for a given flight schedule (Barnhart & Cohn, 2004). Maintenance scheduling is seen as a feasibility problem and it is typically considered after the aircraft routing has been determined (Barnhart, et al., 1998), (Cohn & Barnhart, 2003). This sequential approach can cause that real AMRP instances to have no feasible solutions and when they exist, they can only be found in a range of several hours of computational time. In addition, an optimal routing solution of a flight schedule that is not the optimal one is also a suboptimal solution of the whole problem.

In the context of airline scheduling, Gopalan & Talluri heuristically solve four-day AMRP in polynomial time, and they could identify flights which must be shifted to another fleet to achieve maintenance feasibility (Talluri, 1998), (Gopalan & Talluri, 1998). However, they start with daily aircraft routes, called lines-of-flight (LOFs), which have to be previously determined and fixed. Other approaches determine LOFs and solve heuristically the maintenance routings (Kabbani & Patty, 1992), (Feo & Bard, 1989).

1.3 Maintenance constraints

Maintenance constraints vary depending on the airline, the aircraft, the maintenance program implemented, etc. Talluri describes four types of maintenance for aircraft (Talluri, 1998). They go from infrequent major overhauls to minor visual inspections every few days. Other type of more frequent minor maintenance activities such as cleaning, fueling, inspecting are performed during the turnaround time, together with unloading and loading of passengers.

Maintenance regulations established by the FAA in US are described in (Talluri, 1998), (Gopalan & Talluri, 1998), (Qi, et al., 2004). In particular, they rule conditions about the frequency of maintenance operations (e.g. every maximum number of flying hours, maximum number of landings, maximum number of calendar days, etc.).

A deadhead at a maintenance station lasting at least the minimum maintenance time is referred as a maintenance opportunity (MO) (Cordeau, et al., 2001). Though a maintenance activity can or cannot actually be performed during a MO (Talluri, 1998). The specific aircraft maintenance constraint considered in this paper establishes that any aircraft route must provide a maintenance opportunity at least every certain number of days.

We consider a simplification of this constraint: We assume that all overnight deadheads satisfy this maintenance duration requirement, and there is only one maintenance

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station, which is called the base. In practice, these maintenance operations are taken during nights, where none or very few flight legs are scheduled, and the number of maintenance stations per fleet type is small. These assumptions are not strong, and they are commonly used (Talluri, 1998), (Gopalan & Talluri, 1998). These considerations allow us rephrasing the aircraft maintenance constraint considered in the model as: Any aircraft route must spend at least one night at the base every *nightsout* +1 consecutive day.

2 FORMULATION

In this paper a different formulation for the AMRP is presented. Before presenting it, we first provide a description of the underlying network used to model the problem, where nodes represent the legs, and each leg of the network is a candidate to be included in the final flight schedule.

2.1 Network description

The network used is sometimes known as a *connection network* (also leg-on-node or task network) (Desaulniers, et al., 1997), (Gronksvist, 2005), (Haase & Friberg, 1997). This kind of graph is useful for generating cycling timetables that are repeated for a period of time. Usually the flight schedule is one week period as it is the case of most U.S and Latin American airlines (Barnhart, et al., 1998).

The network $G = (V,A)$ used is an acyclic directed graph. Let $N = \{1,2,\dots, n\}$ be the set of nodes representing the (flight) legs. A leg i is defined by: (sd_i, sl_i, el_i, st_i, et_i) where: (sd_i) is the starting day of leg i, (sl_i) is the departure city of leg i, (el_i) is the arrival city of leg i, (st_i) is the starting time of leg i, and (et_i) is the ending time of leg i. When nodes do not actually represent a flight leg, they are called “null legs”.

Arcs $E = \{(i, j) \mid i, j \text{ compatible}, \ i \in N \ , \ j \in N\}$ in the graph represent only legal connections between two nodes. An arc (i,j) is legal if nodes i and j are compatible and it satisfies individual constraints, if there are any. Nodes i and j are said to be compatible if $st_j \geq et_i$ and $sl_j = el_i$.

A depot $d = 1,\dots|\Delta|$ is an arrival or departure location where a plane is allowed to stay overnight. Let $\Delta = \{s(d), \forall d\}$ and $\Delta' = \{t(d), \forall d\}$ be the set of depot nodes (null legs) where an aircraft route can start and end respectively (i.e. source and sink nodes for each depot). Let d = 1 represents the base depot, where the airline has its base for operations and maintenance.

Let $O(d) \subseteq \{(s(d), j) | s(d) \in \Delta, j \in N : (s(d), j) \text{compatible}\}$ be the set of arcs connecting the depot $s(d)$ with all its compatible legs. These arcs are also called pull-in arcs. A node j is compatible with depot $s(d) \in \Delta$ if $sl_j = d$ (i.e. legs departing from d). Let $O = \cup_{d \in \Delta} \{O(d)\}$.

Similarly, let $D(d) \subseteq \{(i, t(d)) | t(d) \in \Delta', i \in N : (i, t(d)) \text{compatible}\}$, be the set of arcs connecting the legs with their compatible depot $t(d)$. They are also called pull-out arcs. A node i is compatible with depot $t(d) \in \Delta'$ if $el_i = d$. Let $D = \cup_{d \in \Delta'} \{D(d)\}$.

In addition, source and sink nodes s and t, respectively, are added to the network. Finally, let $\Lambda = \bigcup_{\forall d} \{(s, s(d)), (t(d), t)\}$ be the set of arcs connecting departure-depot and end-depot nodes with s and t, respectively. Now, we can define the graph $G = (V, A)$ with nodes $V = N \cup \Delta \cup \Delta' \cup \{s, t\}$, and arcs $A = E \cup O \cup D \cup \Lambda$ as it is shown in Fig. 2 .

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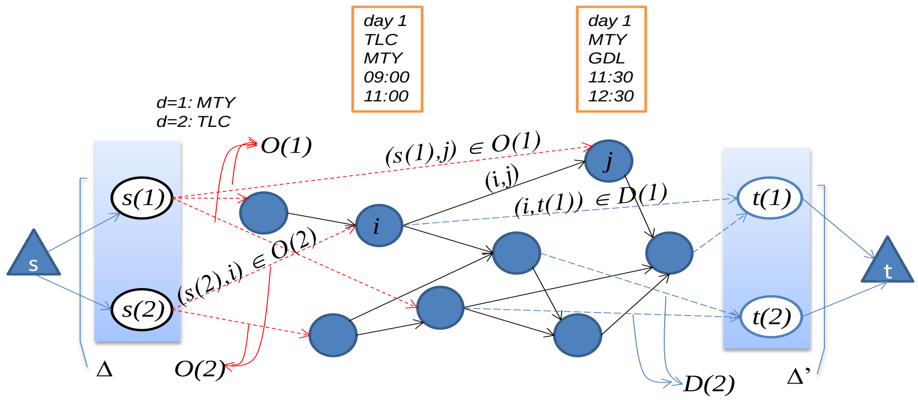

Fig. 2 Connection network G = (V,A) used for generating cycling timetables

2.2 The proposed AMRP model

The model (1)-(6) is used to generate each route. The AMRP is formulated as a Longest Path Problem with side constraints (e.g. maintenance constraints) over the network *G* = (*V,A*). Binary decision variables y_{ij} define if the route visits leg *i* immediately after it visits leg *j* for any arc in *A*.

The objective function (1) maximizes the sum of profits p_{ij} associated with each *(i,j)* in the routing solution. Profit data is assumed to be a given function, usually both node and arc dependent. Equations (2) ensure that the number of arcs entering into a node is the same as those leaving from it. Equations (2) and (4) ensure that the solution corresponds to one complete and cyclic route *s(d)* – *t(d)* . Constraints (5) correspond to the maintenance ones, and constraints (6) impose the integer condition on the arcs between legs. Integrality condition is enough to guarantee binary solutions when $0 \le U_{ij} \le 1$ (e.g. one fleet).

The logic for the maintenance constraints (5) is as follows: Given a weekly time horizon, $nightsout \in [0,1,\ldots,6]$. (e.g. $nightsout = 0$ means that each aircraft must spend every night at the base). Usually this parameter is set to 2 or 3 to guarantee satisfying the governmental regulations (Gopalan & Talluri, 1998). For example, let us set $nightsout = 3$, which is the value used by Airbus 320® fleets. This means that the aircraft must spend one night at base at least once during each four consecutive days. This reasoning must hold for every day of the week, so 7 constraints are needed.

Let $u = 1,\ldots 7$ and $v = 1,\ldots 7$ be two day-index sets, where u refers to the day for which the constraint applies, and v refers to the elements of each constraint. The maintenance constraints (5) say then that if an aircraft spends the night at base on Sunday, for example, then it must spend the night at base at least once between Monday and Thursday. If the aircraft overnights at the base on Monday, it will do it again at least once by Friday, and so on.

Thus $\Theta = \{\theta_{u,v}\}$ is the 7 by 7 coefficient matrix for constraints (5) where $\theta_{uv} = 1$ if overnight arcs on day v are considered for the constraint of the day u. $\theta_{uv} = 0$ otherwise. Θ is then a band matrix with *nightsout* +1 bandwidth. E_v^B is the set of overnight arcs at base during night. An overnight arc is an arc *(i,j)* with $sd_j > sd_i$.

(AMRP Model):

$$Z_{AMRP} = max \sum_{(i,j)\in A} p_{ij} y_{ij} \tag{1}$$

Subject to

$$\sum_{j|(i,j)\in Y} y_{ij} - \sum_{j|(j,i)\in Y} y_{ji} = 0 \qquad \forall i \in N \cup \Delta \cup \Delta' \tag{2}$$

$$\sum_{s(d)|(s,s(d))\in Y} y_{s,s(d)} = 1 \qquad \forall s(d) \in \Delta \tag{3}$$

$$\sum_{t(d)|(t(d),t)\in Y} y_{t(d),t} = 1 \qquad \forall t(d) \in \Delta' \tag{4}$$

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(5) $\sum_{v} \theta_{u,v} * \sum_{(i,j)\in E_v^S} y_{ij} \geq 1 \quad \forall u$

(6) $y_{ij} \leq U_{ij}, \text{integer} \quad \forall (i,j) \in A$

3 SOLUTION METHOD

The solution of the model identifies the route that maximizes profit out all of the possibilities in the network. We start with a large number of possible legs and their estimated profits to the airline. These profits are assumed to be given, known, and fixed for a flight with a specific departure time.

This method does not start with any base. It could be seen as an incremental one in the sense that the final schedule is obtained by the routes found one at the time.

Two assumptions are used. The first one is that the network structure is fixed, not a result of the flight schedule. This is supported by the fact that the structure is more a strategic long term decision than an operational one. The second one is that the network is not fixed every time the AMRP is solved. It changes, as it starts as a very large network that shrinks every time a route is found. Thus, solving the AMRP is just a sub-problem of the whole approach.

3.1 The general approach

The general problem of defining the complete flight schedule together with the aircraft routing is solved by using a proposed heuristic algorithm that solves one AMRP at a time. Each route will give the maximum profit to the airline while satisfying maintenance constraints. The number of routes is limited by the fleet size. In this way the final flight schedule is simultaneously identified together with the aircraft routes while satisfying maintenance requirements.

This method helps dealing with some of the problems that arise in the flight scheduling process: a) Feasibility: the inclusion of any flight leg in a route establishes its feasibility. The definition of flight legs and departure times is given by the routes found. There is no need of a specific or additional formulation for the flight scheduling problem. b) There is no need of pre-established partial solutions (e.g. forcing the inclusion of specific legs), or of base schedules; and c) Dynamic profitability of the flights: since the profit of a flight depends not only on the demand but in the market service, the greedy algorithm allows profits updating at every time a new route is found.

Since the flight schedule must be defined several months in advance, having the routes at the same time allows the airline to know the available opportunities for maintenance, and the information needed for crew scheduling would be ready in advance to facilitate the operation. In addition, this methodology allows identifying those legs that could not be included in the flight schedule with the fleet size given, and that could be either rescheduled on another aircraft fleet or not be flown at all.

3.2 Heuristic algorithm to solve the problem

The Airline Flight Schedule Design and Maintenance Routing Algorithm (AFS-MRA) is proposed. It is a greedy heuristic that identifies, for a single fleet, the flight schedule and simultaneously defines the aircraft routes considering both maintenance constraints and fleet size, given a large number of potential flight legs to be part of the flight schedule. AMRP, as a longest path problem with side constraints, can be solved by any of the techniques mentioned in section 1.1. Its solution constitutes one route satisfying the maintenance constraints. Hence, a cycling process is needed, until one of following three events happen: all legs are part of a route, *F* routes have been identified or the AMRP is found to be infeasible. In this way, the fleet size constraint is not explicitly included in the AMRP model.

Once a route is identified, all variables related with the legs covered in the routing solution are set to zero and the network is then reduced for the next iteration. In other words, for each leg *j* covered in the previous AMRP solution, the upper bounds U_{ij} of the set of arcs arriving to *j* is set to zero, decreasing the size of the problem at any iteration. This algorithm is summarized in Fig. 3 . Additionally, after solving the model any profit's updating of the remaining flight legs or the inclusion of customized rules to delete more

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arcs from the network can be implemented. Those alternatives are not considered in this paper.

This solution algorithm is a heuristic one, in the sense that it does not guarantee to cover all flight legs in the network (which is neither desired). Thus, the final flight schedule to be covered by the fleet will be in fact defined by the routing solutions satisfying maintenance constraints and fleet size capacity.

4 NUMERICAL EXPERIMENTATION

4.1 Test data

This approach has been evaluated with real-world data from three Latin American airlines. The main input data of these instances is a timetable of a set of potential flights to be covered per fleet. Different instances from the same airline correspond to either a timetable per fleet or real itineraries from different operational periods. Test data were obtained as follows:

- *Int1*, *Int2*, and *Int3*. Real flight schedule flown on January, 2006 (ABC Aerolíneas, 2006), January, 2007 (ABC Aerolíneas, 2007), and April, 2007 (ABC Aerolíneas, 2007), respectively. Some additional actual operational data were available to run *Int3*.
- *Cop1*, *Cop2*, *Cop3*. Real itineraries flown on January, 2007 (Copa Airlines, n.d.).
- *Vol1*. Real flight schedule flown on September, 2006 (Volaris, 2006).
- *Vol2*. Based on a data set given by an airline. It contains 33 daily possibilities for each origin-destination option included in *Vol1*.

Table 1 describes these test instances in terms of the available fleet size (AF), the number of destinations (ND) and the number of depots $|\Delta|$.

Begin
  $r \leftarrow 0$
  feasible $\leftarrow$ true
  $ub_{ij} \leftarrow 1 \ \forall (i, j) \in A$
  while $((\exists ub_{ij} \neq 0 \ \forall j : (i,j) \in E) \text{ and } (r < F) \text{ and feasible} = \text{true})$
    solve **model AMRP**.
    if feasible, Let y^*_{ij} be the optimal solution
      set $\Psi = \{y_{ij} : y^*_{ij} \geq 1\}$
      $r \leftarrow r + 1$
      route (*r*)= $\{i : y_{ij} \in \Psi\}$
      for all $i \in N$
        if $y_{ij} = 1$ for some $j : (i, j) \in A$
          then $U_{ij} \leftarrow 0$ for all $j : (i, j) \in E$
        end if
      end for
    else
      feasible $\leftarrow$ false
    end if
  end while
end

Fig. 3 Aircraft Flight Schedule Design and Maintenance Routing Algorithm (AFS-MRA)

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Table 1. Description of the data sets used to validate the proposed the AFS-MRA

Test Sets	AF	ND	$\|\Delta\|$	$\|N\|$	$\|E\|$	$\|A\|$	t_1 (sec)
Int1	3	4	3	170	995	1,309	1.5
Int2	7	13	5	344	3,442	3,990	2.7
Int3	7	13	5	340	7,247	7,789	4.0
Cop1 -738	4	9	9	98	310	520	1.1
Cop2 -E90	4	13	8	182	821	1,151	1.4
Cop3 -73G	20	25	20	560	8,483	9,533	4.4
Vol1	4	7	7	239	1,339	1,831	1.7
Vol2	7	6	6	3,693	316,371	323,769	194.6

4.2 Pre-processing and graph construction

A network reduction or pre-processing was done to significantly decrease the number of arcs, under the additional following assumptions:

- Arcs between legs starting the same day with duration greater than $b_{\max}$ should not be part of any solution, so they are deleted, and
- Arcs between legs starting at different days and lasting more than $B'_{\max}$ should not be part of any solution, (e.g. It is not allowed for an aircraft to be idling for more than, say $B'_{\max} = 24$ hours).

These actions can reduce the network up to about 2% of its size eliminating unwished solutions such as idle route sections of several days in the middle of the week. However, shorter routes are allowed (e.g. idling at the beginning or at the end of the horizon plan).

To provide an idea of the sizes of the problem, the three last columns of Table 1 show the number of arcs between legs ($|E|$), the total number of arcs generated ($|A|$) when the turnaround is 25 minutes, and the computational time, in seconds, required to construct the graph G (t_1), respectively.

4.3 Parameters setting

In order to facilitate comparisons, the same operational parameters values were used in all of the test instances, except for the case of Int3 which contains operational data given by the company. a) Parameters $b_{\max}$ and $B'_{\max}$ were set to 6 and 24 hours respectively in all instances, except in *Int3* where the values were 16 and 48 respectively. b) The network contains only point-to-point flight legs. The time horizon is a week. c) Flight times correspond to local times at the base. d) Not all destinations are depots, but all depots are destinations. e) *Nightsout* is set to 3. f) Minimum maintenance time for a MO is set to 360 minutes. g) The profit function in the AMRP model is assumed to be proportional to the flight lengths. p_{ij} corresponds to the flight duration of leg I, and it is kept fixed.

4.4 Model verification and validation

The models were written in GAMS IDE ® version 2.0.34.19, and solved using ILOG CPLEX® version 10.1. Results were evaluated from two perspectives: time to reach a solution and validity of the results.

Time to reach a solution: Aiming to evaluate if the solution algorithm produces results within reasonable times using standard type of computers for real case instances, all the tests presented in table 1 were run using a MacPro 2 Dual Core Intel Xeon 4 Processors 2.66GHz, 4 GB RAM computer.

Results are evaluated in terms of CPU solution time (t_2) and the number of legs included in the routes (N'). t_2 includes the time required to create the network and to obtain all the routes. Table 2 shows the results obtained by applying the AFS-MRA for all the test sets described in 0 and for different fleet sizes (F). This value F also corresponds to the number of routes and the

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number of times that the cycle of the AFS-MRA is successfully solved for each instance. It also shows the objective function (*OF*), which is given by the sum of legs' flight time in the routes, and the aircraft usage average percentage *(% usage*).

Table 2 shows that for the instances tested, t_2 was less than a minute for the typical cases and less than 25 min for the largest case scenario (more than 3000 legs for airline companies with less than 10 aircrafts).

Fig. 4 shows that t_2 depends not only on *N* but on the network. Relationships vary from nice linear relationships for strong hub-and-spoke networks such *Cop* to exponential relations for dense and short-distance networks as *Vol.*

Aiming to verify that the AMRP proposed model was implemented properly, the routes obtained in every single run were checked that they satisfy the maintenance constraints and the other ones specified in the model. Then the flight schedule is established as the list of legs included in the routes.

Table 2. AFS-MRA results for the data set specified in table 1

Test sets	*F*	|*N*|	|*N´*|	|*A´*|	t_2 *(sec)*	*OF*	*% Usage*
Int1	3	170	156	1,143	2.5	15,940	53
	4	170	164	1,251	2.4	16,640	41
Int2	6	344	294	2,703	5.7	35,350	58
	7	344	326	2,766	6.1	35,625	50
Int3	7	340	340	7,789	3.5	36,485	52
Cop1	4	98	96	498	1.7	16,898	42
Cop2	4	182	110	597	2.5	17,811	44
	7	182	140	840	2.7	21,048	30
Cop3	20	560	379	4,701	29.5	98,169	49
	25	560	485	7,251	39.1	110,747	44
	28	560	506	7,717	45.4	113,337	40
Vol1	6	239	217	1,512	3.5	36,750	61
	7	239	**223**	1,580	3.7	37,455	53
Vol2	**7**	3,693	**368**	4,780	853.1	55,335	78
	10	3,693	522	9,277	1274.1	78,110	77

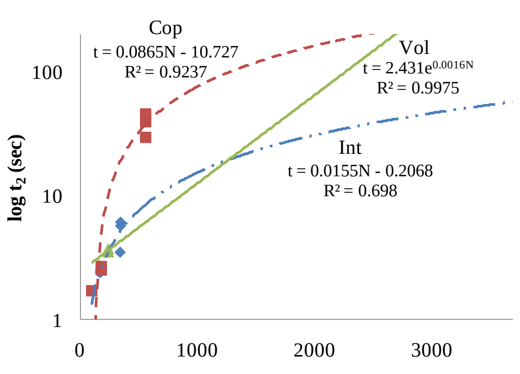

Fig. 4 CPU time (t_2) as function of number of legs of the instance (N)

To validate that the proposed AFS-MRA is well formulated and that it actually leads to better solutions or at least similar solutions than the traditional sequential approach, several alternatives were considered. First at all, the total cost of the solution per unit of flight time ($\overline{C}$) is used as a criterion of performance.

Then it was attempted to compare results of the AFS-MRA against the ones reported by any airline planning commercial software. However authors did not have access to any of this type of software. Then, it was proposed to develop an in-house one (from now on named as VSP model) where the vehicle scheduling problem has to be formulated for the same connection network *G*.

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This is the simplest option that will give the lowest possible bound in cost.

The VSP model is presented in equations (7) to (15). The objective function (7) minimizes the sum of costs associated with each connection in the solution. Assignment constraints are (8) and (9) One arc must begin and one arc must end at each leg. Constraints (10) and (11) ensure flow balance at depot nodes, and (12) do the same for each depot. Constraint (13) limits the number of routes to the fleet size, since $y^*_{s,s(d)}$, for each $s(d) \in \Delta$ gives the number of routes leaving each depot *d*. Finally, constraints (14) and (15) impose the binary condition on the arcs between legs, and integer condition on the arcs connecting depots. This model was implemented in GAMS.

For comparison purposes, the cost unit used is time. c_{ij} represents the deadhead between legs *i* and *j*. For a fixed horizon time, the flight time to be maximized in AMRP can be related to the deadhead time to be minimized in VRP. One value is the complement of the other for a given period.

(Model VSP):

$$z_{VSP} = \min \sum_{(i,j)\in A} c_{ij} y_{ij} \tag{7}$$

Subject to

$$\sum_{j:(i,j)\in A} y_{ij} = 1 \qquad \forall i \in N \tag{8}$$

$$\sum_{i:(i,j)\in A} y_{ij} = 1 \qquad \forall j \in N \tag{9}$$

$$\sum_{j:(s(d),j)\in A} y_{s(d),j} - y_{s,s(d)} = 0 \qquad \forall s^d \in \Delta \tag{10}$$

$$\sum_{i:(i,t(d))\in A} y_{i,t(d)} - y_{t(d),t} = 0 \qquad \forall t^d \in \Delta' \tag{11}$$

$$y_{s,s(d)} - y_{t(d),t} = 0 \qquad \forall d \tag{12}$$

$$\sum_{d} y_{s,s(d)} \le F \tag{13}$$

$$y_{ij} \in \{0,1\}, \forall (i,j) \in E \tag{14}$$

$$y_{ij} \in \mathbb{N}, \forall (i,j) \in A \setminus E \tag{15}$$

Two different types of comparisons were done for every instance specified in table 2:

- *Full set of flights*: The network considers the original data set with all the flights to be scheduled. VSP model routes all the flights. However AFS-MRA chooses does not necessarily include 100% of these flights.
- *Partial set of flights*: with the flight set chosen by the AFS-MRA, a smaller connection network is established to run the VSP model.

The VSP model has two major simplifications:

- The VSP model allows finding sequences starting from different depots. However, the VSP model guarantees that the same number of sequences leaves and arrives at each depot, but it does not guarantee that all sequences are routes (i.e. they start and end at the same depot).

- The VSP model does not include any maintenance constraints. This simplification was necessary to be done since airline companies did not provide such information.

Given the simplicity of the VSP model, this one provides the best lower bounds to our AFS-MRA, and it is used only for comparison purposes, since results from this VSP model can still provide an indication of the performance of the new AFS-MRA. Clearly results from the VSP model without maintenance will provide lower bounds for the cost to the case where it is considered.

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Table 3. Comparison of VSP and AFS-MRA results

Instance	F	AFS-MRA	VSP			
			Full set of legs		Partial set of legs	
		$\bar{C}_{AFS-MRA}$	$\bar{C}^{F}_{CVS}$	R_F	$\bar{C}^{P}_{CVS}$	R_P
Int1	3	34.3	42.3	0.81	34.3	1.00
Int1	4	50.1	37.8	1.33	30.9	1.62
Int2	7	36.3	42.6	0.85	36.3	1.00
Int2	6	32.9	42.6	0.77	32.9	1.00
Int3	7	11.7	11.7	1.00	11.7	1.00
Cop1	4	45.8	45.7	1.00	45.8	1.00
Cop2	4	42.5	58.1	0.73	42.5	1.00
Cop2	7	75.6	58.1	1.30	61.2	1.24
Cop3	20	34.5	45.7	0.75	34.5	1.00
Cop3	25	41.5	45.7	0.91	41.5	1.00
Cop3	28	47.5	45.7	1.04	45.4	1.05
Vol1	6	23.8	28.9	0.82	23.8	1.00
Vol1	7	30.9	28.9	1.07	30.7	1.01

Table 3 and Fig. 5 compare results from both models. $\bar{C}_{AFS-MRA}$ and $\bar{C}_{VSP}$ are defined as cost per flight time obtained with the AFS-MRA and with the VPS model, respectively. For convenience R_F and R_P are defined as the ratio of cost per flight time obtained with the AFS-MRA over the cost per flight time obtained with the VPS model for the cases of full set of flights and partial set of flights, respectively. Therefore, R_F or R_P smaller than one mean that the AFS-MRA generates a solution less costly than the one provided by the base model (VSP model without maintenance).

For the *case of partial set of legs* (same flight sets), table 3 and figure 5 show that for most of the instances, the AFS-MRA generates a solution at the same cost per flight time that the VSP model ($R_P \approx 1$), with the benefit that it already includes maintenance constraints. Therefore, it can be concluded that, for these cases, the AFS-MRA leads to better solutions that the traditional sequential approach (VSP model considering maintenance).

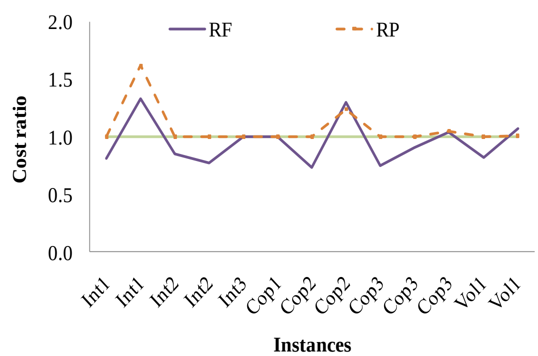

Fig. 5 Ratios of the total cost per flight times of the AFS-MRA and the cost VSP solution. R_F: full set of flights, R_P: partial set of flights.

Table 2 shows that there are two cases where R_P >1. They are highlighted in tables 2 and 3. Table 2 shows that they correspond to instances where the usage percentage of the aircrafts is low compared to most of the instances. This value for the *Int1* instance with F=4 and N=140 is affected by the fact that the last aircraft covers 8 additional flights since there were only 14 flights left in the network. For the *Cop2* instance, with F=7 and N=180, the last 3 aircrafts cover 30 out of 72 flights left in the network. On the other hand, the last two instances with a large N (*Vol2*) give an average usage percentage of 77%, which is 67%

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better than the average of all other instances. This means that the performance of the algorithm increases when there is a large set of flights left in the network.

For the case of *full set of legs* (different set of flights) table 3 and figure 5 show that, except for the two previous instances, the AFS-MRA generates a solution at lower of equal cost per flight time that the VSP model (R_F =1)

Having R_P or R_F >1 does not mean that the proposed AFS-MRA does not provide better solutions than the traditional sequential approach. It means that it is required to include the maintenance constrains within the VSP model to have results that are truly comparable to the ones reported by the AFS-MRA.

5 CONCLUSIONS AND FURTHER WORK

In this work, the paradigm of defining first the flight schedule and, in a later stage, to solve the aircraft routing is changed for the case of single fleet airline companies. Here it is presented a model to easily solve the routing problem with maintenance constraints for single fleet airlines, and simultaneously gives the schedule design.

Initially the formulation of this optimization problem is presented as a Longest Path Problem with side constraints. It is based on a connecting network where a flight schedule is defined as the set of flight legs that include all routes to be flown. Each route satisfies maintenance requirements.

Then a solution algorithm is proposed to solve this integrated problem. It is a greedy heuristic that starting from a given large number of potential flight legs considered by the marketing division, it identifies at each cycle the flight leg that provide the maximum profit, defining simultaneously flight schedule and aircraft routing at the same time that consider maintenance constraints and fleet size. This solution algorithm is a heuristic one, in the sense that it does not guarantee to cover all flight legs in the network and that the profit obtained, at each iteration, does not necessarily increase.

The model was tested using 7 instances extracted from real-world timetables of three Latin American airlines, and one hypothetical instance given by one of them.

It was found that the model can be solved using standard type of computers and that the CPU time can increase from linearly to exponentially, depending not only on the number of flights being considered but on the network itself. Better performance is found for strong hub-and-spoke networks. This time is less than a minute for the typical cases of single fleet airlines and it is less than 25 min for the largest case scenario of more than 3000 legs for an airline company with less than 10 aircrafts.

To validate the proposed model, results were compared to the results of a standard sequential model (VSP model) that does not include maintenance constrains.

Two cases were considered: full and partial set of flights. In the first case the VSP model routes 100% of the flights while the proposed model can exclude some of them. In the second case the sequential model is run with the flight legs routed by the AFS-MRA model.

Results were compared in terms of the cost per flight time. It was found that for most of the instances, the AFS-MRA generates a solution at the same o lower cost per flight time that the VSP model. Therefore, it can be concluded that, for these cases, the AFS-MRA leads to better solutions that the traditional sequential approach (VSP model considering maintenance).

Results showed that computational time increases from linear to exponential fashions with respect to N, depending on the network structure of the airline, and the performance of the algorithm, measured as usage percentage, increases with *N*.

For these cases it is required additional work to include the maintenance constrains within the VSP model and therefore obtains results that are truly comparable to the ones reported by the AFS-MRA.

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Finally, the greedy algorithm that is proposed here allows a profit data updating for the potential legs remaining in the network every time a new route is found. This updating would allow take in consideration that profit actually depends on other flights offered by the airline and even its competitors. As further work, it is expected this profit data updating will improve the quality of the routing solutions.

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