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OPTIMAL REPLENISHMENT POLICY FOR DETERIORATING INVENTORY WITH TRADE CREDIT FINANCING, CAPACITY CONSTRAINTS AND STOCK-DEPENDENT DEMAND

[1]Nita H. Shah and Arpan D. Shah

Department of Mathematics, Gujarat University, Ahmedabad-380009, India
Corresponding Author: [1]Prof. Nita H. Shah E-mail address: nitahshah@gmail.com

ABSTRACT

This research aims at optimizing the replenishment cycle time when units in inventory deteriorate at a constant rate under a permissible delay in payments when demand is stock-dependent. It is assumed that own warehouse capacity is limited and excess inventory is stored in rented warehouse. The unit holding cost in rented warehouse is higher than that in the owned warehouse. The deterioration rates of items in owned warehouse and rented warehouse are different. A decision policy is worked out to make a choice of own warehouse or hire the rented warehouse. The uniqueness of the optimal solution is derived. Finally, numerical examples are presented to validate the proposed problem. Sensitivity analysis is carried out to derive managerial insights.

KEYWORDS:
Inventory, EOQ, Stock-dependent demand, Two-warehouse, Deterioration, Trade Credit

1. INTRODUCTION

In business transaction, the offer of allowable credit period to settle the accounts against the purchases made is considered to be an effective promotional tool. This tool stimulates the demand, attracts customers etc. Goyal (1985) derived an EOQ model under the condition of a permissible delay in payments. Huang (2004) revisited Goyal (1985)'s model and obtained the efficient algorithm to determine the optimal cycle time. Manna and Chaudhuri (2005) studied optimal ordering policy for deteriorating item under a "net credit" policy using DCF approach. Manna *et al.* (2008) discussed inventory model when the supplier offers a fixed credit period to the retailer and units in inventory deteriorate with respect to time. They established that the supplier incurs more profit when credit period is greater that the replenishment cycle time. Shah *et al. (*2010) gave an up-to-date review article comprising of available literature on trade credit.

From an economic prospective, credit policies are considered to be an alternative to price discount to favor larger orders, so the question is where to stock this order. The most of the references sited in Shah *et al. (*2010) assumed that the retailer owns a single warehouse with unlimited capacity. However, any warehouse has finite capacity.

Hartely (1976) developed an inventory model comprising of two warehouses viz. owned warehouse (OW) and rented warehouse (RW). The units more than the fixed capacity were stored in the rented warehouse. The holding cost of an item in the RW was assumed to be higher than OW. Sarma (1993) assumed infinite production rate and analyzed optimal strategies for two warehouses. Goswami and Chaudhari (1992) allowed shortages when demand is increasing linearly. These models do not consider deterioration of item, Sarma (1987) first formulated a two-warehouse model for deteriorating items with an infinite replenishment rate and shortages. Some other relevant articles by Pakkala and Acharya (1992-a, 1992-b),Hiroaki and Nose (1996), Benkherouf (1997), Yang (2004), Zhou and yang (2005), Lee (2006), Yang (2006).

Shah and Shah (2012) discussed variants of two-warehouse inventory models in fuzzy enviornment.

Levin *et al*. (1972) quoted that "large piles of goods attract more customers". This was termed as stock-dependent demand. Urban (2005) obtained optimal policies when demand is

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stock-dependent. Zhou and Yang (2005) developed a two-warehouse model with a stock-dependent demand and consideration of transportation cost. Roy and Chaudhuri (2007) developed an inventory model when demand is stock-dependent and planning horizon is finite. The effects of inflation and time value of money are incorporated to optimize objective function. They allowed complete back-logging. Roy and Chaudhuri (2012) discussed an economic production lot size model for price-sensitive stock-dependent demand when units in inventory are subject to constant rate of deterioration.

Liao and Huang (2010) analyzed replenishment policy for deteriorating items with two-storage facilities and a permissible payment delay. This study aims to develop optimal ordering policy for deteriorating items with two-warehouse under stock-dependent demand and credit financing. The deterioration rates of items in two-warehouses are different. The unit holding cost in rented warehouse (RW) is higher than that in owned warehouse (OW). The objective is to maximize the total profit. The analytic results are derived to establish the existence and uniqueness of the cycle time. The theorems are deduced to determine the optimal cycle time. The numerical examples are given to illustrate these theorems. These results will help the decision maker to decide "Whether or not to rent RW?" to stock more items to maximize annual profits.

2. Notations and Assumptions

The following notations and assumptions are adopted to develop proposed mathematical model.

2.1 Notations

OW Owned warehouse

RW Rented warehouse

A The ordering cost per order

$R(I_k(t))$ The stock dependent demand rate $R(I_k(t)) = \alpha + \beta I_k(t)$,where $\alpha > 0$ denotes scale demand; $0 < \beta < 1$ denotes stock-dependent parameter; and $I_k(t)$ denotes inventory level at any instant of time t . where $k = o$ or r

W The finite storage capacity of OW

P The selling price per unit

C The purchase cost per unit, with $C \le P$

h_o The holding cost per unit time in OW

h_r The holding cost per unit time in RW, $h_r \ge h_o$

θ_o The deterioration rate OW, $0 < \theta_o < 1$

θ_r The deterioration rate RW, $0 < \theta_r < 1$ and $\theta_r > \theta_o$

T The cycle time (a decision variable)

Q The purchase quantity (a decision variable)

T_r The time at which inventory depletes to zero in RW

T_o $\frac{1}{\eta_o}\ln\left(1+\frac{\eta_o w}{\alpha}\right)$, time at which inventory depletes to zero in OW, where $\eta_o = \theta_o + \beta$

M The credit period offered by the supplier $M < T_o$

I_c Interest charged per unit per annum

I_e Interest earned per $ per annum

Z(T) The annual net profit per unit time.

2.2 Assumptions

(1) The two-warehouse inventory system stocks single item.
(2) The planning horizon is infinite.
(3) Lead-time is zero or negligible. Shortages are not allowed.
(4) The OW has a fixed capacity of W-units.
(5) The RW has unlimited capacity.
(6) The items in RW are consumed first.
(7) The holding cost and deterioration cost in RW are higher than those in the OW.
(8) The supplier offers a credit period. The retailer earns interest at the rate I_e on the sales revenue during allowable credit period. At the end of the credit period, the account is settled. On the unsold stock, the retailer has to pay interest charges at the rate I_c (Shah (1993).
(9) $I_o(t)$ denotes the inventory level at time $t \in (0, T_r)$ in the OW, in which the

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inventory is depleting due to deterioration of the item and stock-dependent parameter of inventory. $I_r(t)$ denotes the inventory level at time $t \in (0, T_r)$ in the RW, in which the inventory is decreasing to zero due to stock-dependent demand and deterioration of the item. $I(t)$ denotes the inventory level at time $t \in (T_r, T)$ where the inventory level drops to zero due to demand and deterioration of the item.

3. MATHEMATICAL MODEL

The retailer has to decide whether it is advantageous RW to stock more items to maximize annual net profit or not when offer trade credit is available. Obviously if $Q \le W$, then there is no need of RW. Otherwise, W units are stocked in OW and remaining in RW. This suggests that we need to discuss following two scenarios: (A) the single-warehouse inventory system, and (B) the two-warehouse inventory system. If we denote $T_o = \frac{1}{\eta_o}\ln\left(1+\frac{\eta_o w}{\alpha}\right)$, (FIG - 1) the inequality $Q \le W$ holds if and only if $T_o \ge T$

3.1 Single-warehouse inventory model ($T_o \ge T$) (Fig.1)

The cycle starts with Q units in the inventory system. During the period $(0, T)$ the inventory level depletes in the OW due to stock-dependent demand and deterioration. The rate of change of inventory level at any instant of time t is governed by the differential equation

$$\frac{dI_o(t)}{dt} + (\theta_o + \beta) I(t) = -\alpha, \; 0 < t \le T \qquad (1)$$

Let $\eta_o = \theta_o + \beta$ Using boundary condition $I_o(T) = 0$, the solution of differential equation (1) is

$$I_o(t) = \frac{\alpha}{\eta_o}\left[e^{\eta_o(T-t)} - 1\right], \; 0 \le t \le T \qquad (2)$$

and the order quantity is

$$Q = I_o(0) = \frac{\alpha}{\eta_o}\left[e^{\eta_o T_o} - 1\right] \qquad (3)$$

The different components of annual net profit per unit time of the inventory system are

(1) Sales revenue = $(P - C)\frac{Q}{T}$

(2) Ordering cost = $\frac{A}{T}$

(3) Holding cost in the RW=0 because we need only one warehouse.

(4) Holding cost in the OW

$$= \frac{h_o}{T}\int_0^T I_o(t)dt = \frac{h_o \alpha}{\eta_o^2 T}\left[e^{\eta_o T} - \eta_o T - 1\right]$$

For interest earned and interest charged we need to observe the length M and T.

Two cases may arise

Case 1: $T \le M$

Here, all units are sold off before the allowable credit period. So the interest charges are zero and the interest earned per unit time is

$$\frac{PI_e}{T}\left[\int_0^T R(I_o(t))tdt + Q(M-T)\right]$$

$$= \frac{PI_e \alpha}{T}\left[\begin{array}{l}\frac{1}{2}\left(1+\frac{\beta}{\eta_o}\right)T^2 + \frac{\beta}{\eta_o}\left(e^{\eta_o T} - \eta_o T - 1\right) \\ + \frac{1}{\eta_o}\left(e^{\eta_o T} - 1\right)(M-T)\end{array}\right]$$

Case 2: $M < T$

In this case, the retailer sells the items and pays at M. So during $[0, M]$, the interest earned per unit time is $\frac{PI_e}{T}\left[\int_0^M R(I_o(t))tdt\right]$ and during $[M, T]$, the interest paid at the rate I_c on the unsold stock is $\frac{CI_c}{T}\int_M^T I_o(t)dt$.

The annual net profit per unit time, Z(T) is sales revenue minus sum of ordering cost, holding cost in OW, interest charged plus interest earned. Consequently, the net profit per unit time is

$$Z(T) = \begin{cases} Z_1(T), if\, 0 < T < M \\ Z_2(T), if M \le T \end{cases} \qquad (4)$$

Where, $Z_1(T) =$

$$(P-C)\frac{\alpha}{\eta_o T}\left[e^{\eta_o T} - 1\right] - \frac{A}{T} - \frac{h_o \alpha}{\eta_o^2 T}\left[e^{\eta_o T} - \eta_o T - 1\right]$$

$$+ \frac{PI_e \alpha}{T}\left[\begin{array}{l}\frac{1}{2}\left(1+\frac{\beta}{\eta_o}\right)T^2 + \frac{\beta}{\eta_o}\left(e^{\eta_o T} - \eta_o T - 1\right) \\ + \frac{1}{\eta_o}\left(e^{\eta_o T} - 1\right)(M-T)\end{array}\right]$$

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(5)

$$Z_2(T) = (P-C)\frac{\alpha}{\eta_o T}\left[e^{\eta_o T}-1\right] - \frac{A}{T} - \frac{h_o\alpha}{\eta_o^2 T}\left[e^{\eta_o T}-\eta_o T-1\right] + \frac{PI_e\alpha}{T}\begin{bmatrix}\frac{1}{2}\left(1+\frac{\beta}{\eta_o}\right)M^2 \\ +\frac{\beta}{\eta_o^3}\left(e^{\eta_o T}-(1+\eta_o M)e^{\eta_o(T-M)}\right)\end{bmatrix} - \frac{PI_e\alpha}{T}\left[\frac{CI_c\alpha}{\eta_o^2 T}\left(e^{\eta_o(T-M)}-\eta_o(T-M)-1\right)\right]$$

(6)

Since, $Z_1(M) = Z_2(M)$, Z(T) is well-defined and continuous. The first and second order derivative of $Z_1(T) = Z_2(T)$ are

$$Z_1^{'}(T) = \frac{1}{T^2}\begin{Bmatrix} A+\frac{\alpha}{\eta_o^2}\left[(1-\eta_o T)e^{\eta_o T}-1\right]\begin{bmatrix}h_o-(P-C)\eta_o \\ -PI_e\beta\end{bmatrix} \\ +PI_e\alpha\begin{bmatrix}\frac{1}{2}\left(1+\frac{\beta}{\eta_o}\right)T^2+Te^{\eta_o T}(M-T) \\ -\frac{M}{\eta_o}\left(e^{\eta_o T}-1\right)\end{bmatrix}\end{Bmatrix} \quad (7)$$

$$Z_1^{''}(T) = \frac{1}{T^3}\{-2A\} - \frac{\alpha}{T^3}\begin{bmatrix}\frac{2}{\eta_o}\left(\left(1-\eta_o T\right)e^{\eta_o T}-1\right) \\ +e^{\eta_o T}T^2\end{bmatrix}\begin{bmatrix}h_o \\ -(P-C)\eta_o \\ -PI_e\beta\end{bmatrix} + \frac{PI_e\alpha}{T^3}\begin{bmatrix}\eta_o T^2 e^{\eta_o T}(M-T)-2MTe^{\eta_o T} \\ +\frac{2M}{\eta_o}(e^{\eta_o T}-1)\end{bmatrix} \quad (8)$$

$$Z_2^{'}(T) = \frac{1}{T^2}\left\{A+\frac{\alpha}{\eta_o^2}\begin{bmatrix}\left(1-\eta_o T\right)e^{\eta_o T} \\ -1\end{bmatrix}\begin{bmatrix}h_o-(P-C)\eta_o \\ -PI_e\beta\end{bmatrix}\right\}$$

$$\frac{1}{T^2}\left[\frac{CI_c\alpha}{\eta_o^2}\left[e^{\eta_o(T-M)}(1-\eta_o T)+\eta_o M-1\right]\right]$$

$$+\frac{PI_e\alpha}{T^2}\begin{bmatrix}-\frac{1}{2}\left(1+\frac{\beta}{\eta_o}\right)M^2 \\ +\frac{\beta}{\eta_o^3}\begin{bmatrix}(1-\eta_o T)e^{\eta_o(T-M)} \\ -\left(1-\eta_o T\right)e^{\eta_o T} \\ +\eta_o Me^{\eta_o(T-M)}\left(1-\eta_o T\right)\end{bmatrix}\end{bmatrix}$$

(9) (6)

$$Z_2^{''}(T) = \frac{1}{T^3}\left\{-2A+X\begin{bmatrix}-2e^{\eta_o T}+2\eta_o Te^{\eta_o T}+2 \\ +\eta_o Te^{\eta_o T}-\eta_o^2T^2e^{\eta_o T}\end{bmatrix}\right\} + \frac{1}{T^3}\left\{\frac{CI_c\alpha}{\eta_o^2}\begin{bmatrix}-2e^{\eta_o(T-M)}+2\eta_o Te^{\eta_o(T-M)} \\ -2\eta_o M+2+\eta_o T^2e^{\eta_o(T-M)}\end{bmatrix}\right\}$$

$$+\frac{PIe\alpha}{T^3}\begin{bmatrix}-\left(1+\frac{\beta}{\eta_o}\right)M^2 \\ +\frac{\beta}{\eta_o^3}\begin{pmatrix}-2e^{\eta_o(T-M)}+2e^{\eta_o T}-2\eta_o Me^{\eta_o(T-M)} \\ -2\eta_o Te^{\eta_o T} \\ +2\eta_o^2TMe^{\eta_o(T-M)}+2\eta_o TMe^{\eta_o(T-M)}\end{pmatrix}\end{bmatrix}$$

$$+\frac{PI_e\alpha}{T^3}\frac{\beta}{\eta_o^3}\begin{pmatrix}+\eta_o^2T^2e^{\eta_o T}-\eta_0^3T^2Me^{\eta_o(T-M)} \\ -\eta_o^2T^2e^{\eta_o(T-M)}\end{pmatrix} \quad (10)$$

If $\beta = 0$, equations (4)-(10) are consistent with those given in Liao and Huang (2010). Additionally, if $P = C$, then these equations are some as given in Hwang and Shinn (1997). Using lemma 1 and 2 of Chung et al. (2001), we have $Z_1^{''}(T) < 0$ for all T, and $Z_2^{''}(T) < 0$ for all $T \geq M$ respectively. Hence, we have following theorem.

Theorem 1 Let $T \leq T_o$

1. $Z_1(T)$ is concave on $(0, \infty)$.

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2. $Z_2(T)$ is concave on $[M,\infty)$.
3. $Z(T)$ is concave on $(0,\infty)$.

Also, let

$$\Delta = A+\frac{\alpha}{\eta_o^2}\left[(1-\eta_o M)e^{\eta_o M}-1\right]\begin{bmatrix} h_o-(P-C)\eta_o \\ -PI_e\beta \end{bmatrix}$$

$$+PI_e\alpha\left[\frac{1}{2}\left(1+\frac{\beta}{\eta_o}\right)M^2-\frac{M}{\eta_o}\left(e^{\eta_o M}-1\right)\right]$$

Similar to theorem 2 of Chung et al. (2001), we have following theorem.

THEOREM 2

1. If $\Delta > 0$, then $T^* = T_2^*$.
2. If $\Delta < 0$, then $T^* = T_1^*$.
3. If $\Delta = 0$, then $T^* = T_1^* = T_2^* = M$.

Thus, it is established that for $T \le T_o$, the retailer uses only OW and order $Q^* \le W$.

3.2 Two warehouse inventory model $(T_o < T)$ **(Fig 2).**

$Q > W$ indicates that two warehouse inventory system is involved. Out of Q units received in the beginning of the cycle, W units are kept in the OW and remaining $Q-W$ units are stocked in the RW. During $(0,T_r)$ we first consume items from RW and then from OW. Since both warehouse are having different stocking facilities. The rate of change of inventory level in RW during $(0,T_r)$ can be described by the differential equation,

$$\frac{dI_r(t)}{dt}+\eta_r I_r(t)=-\alpha, \quad 0\le t\le T_r \tag{11}$$

with boundary conditions $I_r(T_r)=0$. Here $\eta_r=\theta_r+\beta$. The solution of equation (11) is

$$I_r(t)=\frac{\alpha}{\eta_r}\left[e^{\eta_r(T_r-t)}-1\right], \quad 0\le t\le T_r \tag{12}$$

During $(0,T_r)$ the rate of change of inventory level, $I_o(t)$ in OW is governed by the differential equation,

$$\frac{dI_o(t)}{dt}=\eta_o I_o(t), \quad 0<t<T_r \tag{13}$$

With initial condition $I_o(0)=W$. The solution of equation (13) is

$$I_o(t)=We^{-\eta_o t}, \quad 0\le t\le T_r \tag{14}$$

During (T_r,T), the rate of change of inventory is governed by the differential equation,

$$\frac{dI(t)}{dt}+\eta_o I(t)=-\alpha, \quad T_r\le t\le T \tag{15}$$

with boundary condition $I(T)=0$. The solution of equation (15) is

$$I(t)=\frac{\alpha}{\eta_o}\left[e^{\eta_o(T-t)}-1\right], \quad T_r\le t\le T \tag{16}$$

The order quantity during the cycle time is $Q=I_r(0)+I_o(0)$

$$=\frac{\alpha}{\eta_o}\left[e^{\eta_r T_r}-1\right]+W \tag{17}$$

Similar to section 3.1, the different cost components of the annual net profit per unit time are

1. Sales revenue = $(P-C)\dfrac{Q}{T}$
2. Ordering cost = $\dfrac{A}{T}$
3. Holding cost in RW=
$$=\frac{h_r}{T}\int_0^{T_r} I_r(t)dt=\frac{h_r\alpha}{\eta_r^2 T}\left[e^{\eta_r T_r}-\eta_r T_r-1\right],$$
$$\eta_r=\theta_r+\beta$$
4. Holding cost in OW =
$$\frac{h_o}{T}\left[\int_0^{T_r} I_o(t)dt+\int_{T_r}^{T} I(t)dt\right]$$
$$=\frac{h_o}{T}\left[\frac{W}{\eta_o}\left(1-e^{-\eta_o T_r}\right)+\frac{\alpha}{\eta_o^2}\begin{pmatrix} e^{\eta_o(T-T_r)} \\ -\eta_o(T-T_r)-1 \end{pmatrix}\right]$$
5. Interest earned is same as given in single ware house case for $M<T$.
6. Interest charged
$$=\frac{CI_c}{T}\left[\int_M^{T_r} I_r(t)dt+\int_M^{T_r} I_o(t)dt+\int_{T_r}^{T} I(t)dt\right]$$
$$=\frac{CI_c}{T}\begin{bmatrix} \dfrac{\alpha}{\eta_r^2}\left(e^{\eta_r(T_r-M)}-\eta_r(T_r-M)-1\right) \\ +\dfrac{W}{\eta_o}\left(e^{-\eta_o M}-e^{-\eta_o T_r}\right) \\ +\dfrac{\alpha}{\eta_o^2}\left(e^{\eta_o(T-T_r)}-\eta_o(T-T_r)-1\right) \end{bmatrix}$$

Hence, the annual net profit per unit time is,

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$$Z_3(T) = (P-C)\frac{Q}{T} - \frac{A}{T} - \frac{h_r\alpha}{\eta_r^2 T}\left[e^{\eta_r T_r} - \eta_r T_r - 1\right] - \frac{h_o}{T}\left[\frac{W}{\eta_o}\left(1-e^{-\eta_o T_r}\right) + \frac{\alpha}{\eta_o^2}\begin{pmatrix} e^{\eta_o(T-T_r)} \\ -\eta_o(T-T_r)-1\end{pmatrix}\right]$$

$$-\frac{CI_c}{T}\begin{bmatrix} \frac{\alpha}{\eta_r^2}\left(e^{\eta_r(T_r-M)} - \eta_r(T_r-M)-1\right) \\ +\frac{W}{\eta_o}\left(e^{-\eta_o M} - e^{-\eta_o T_r}\right) \\ +\frac{\alpha}{\eta_o^2}\begin{pmatrix} e^{\eta_o(T-T_r)} \\ -\eta_o(T-T_r)-1\end{pmatrix}\end{bmatrix}$$

$$+\frac{PI_e\alpha}{T}\begin{bmatrix} \frac{1}{2}\left(1+\frac{\beta}{\eta_o}\right)M^2 \\ +\frac{\beta}{\eta_o^3}\begin{pmatrix} e^{\eta_o T} \\ -(1+\eta_o M)e^{\eta_o(T-M)}\end{pmatrix}\end{bmatrix} \quad (18)$$

Using continuity, we have $I_o(T_r) = I(T_r)$, which gives T_r to be a function of T as

$$T_r = \frac{1}{\eta_o}\ln\left(\frac{\alpha e^{\eta_o T} - \eta_o W}{\alpha}\right) \quad (19)$$

Also,

$$\frac{dT_r}{dT} = \frac{\alpha e^{\eta_o T}}{\alpha e^{\eta_o T} - \eta_o W} > 1 \quad (20)$$

Substituting value of T_r from equation (19) into equation (18), we obtain annual net profit per unit time to be a function of T. The necessary condition for $Z_3(T)$ to be maximum is to set $Z_3^{'}(T)$ to be zero.

Thus we obtain

$$\frac{dZ_3(T)}{dT} = \frac{1}{T^2} f_3(T) \quad (21)$$

where,

$$f_3(T) = A + (P-C)\left[W + \alpha T e^{\eta_r T_r}\frac{dT_r}{dT} - \frac{\alpha}{\eta_r}(e^{\eta_r T_r}-1)\right] + \frac{h_r\alpha}{\eta_r^2}\left[e^{\eta_r T_r} - \eta_r T_r - 1 + \eta_r T(e^{\eta_r T_r}+1)\frac{dT_r}{dT}\right]$$

$$+h_o\begin{bmatrix} \frac{W}{\eta_o}\left(1-e^{-\eta_o T_r}\right) + \frac{\alpha}{\eta_o^2}\left(e^{\eta_o(T-T_r)} - \eta_o(T-T_r)-1\right) \\ -WTe^{-\eta_o T_r}\frac{dT_r}{dT} + \frac{\alpha T}{\eta_o}\left(e^{\eta_o(T-T_r)}-1\right)(1-\frac{dT_r}{dT})\end{bmatrix}$$

$$+CI_c h(T) - PI_e\alpha\begin{bmatrix} \frac{1}{2}\left(1+\frac{\beta}{\eta_o}\right)M^2 \\ +\frac{\beta}{\eta_o^3}\begin{pmatrix} (1-\eta_o T)e^{\eta_o T} \\ -(1-\eta_o T)e^{\eta_o(T-M)} \\ -\eta_o M(1-\eta_o T)e^{\eta_o(T-M)}\end{pmatrix}\end{bmatrix} \quad (22)$$

Where

$$h(T) = \frac{\alpha}{\eta_r^2}\left(e^{\eta_r(T-M)} - \eta_r(T_r-M)-1\right) - \eta_r T\left(e^{\eta_r(T-M)}-1\right)\frac{dT_r}{dT} + \frac{W}{\eta_o}\left(e^{-\eta_o M} - e^{-\eta_o T_r}\left(1+\eta_o T\frac{dT_r}{dT}\right)\right) + \frac{\alpha}{\eta_o^2}\begin{pmatrix} e^{\eta_o(T-T_r)} - \eta_o(T-T_r)-1 \\ -\eta_o T\left(e^{\eta_o(T-T_r)}-1\right)\left(1-\frac{dT_r}{dT}\right)\end{pmatrix}$$

Clearly, both $f_3(T)$ and $Z_3(T)$ have the same sign and domain. Let T_3^* if it exists be the solution of $f_3(T)=0$. Then we claim following theorem.

Theorem 3

1. If $f_3(T_o) > 0, T_3^*$ is the unique cycle time which maximizes $Z_3(T)$ on $[T_o,\infty)$.
2. If $f_3(T_o) \le 0$, then $Z_3(T)$ is decreasing on $[T_o,\infty)$. So, $T_3^* = T_o$.

Proof:

1. Proof follows from Shah (1993) and Thomas and Finney (1996).
2. Proof is similar to that of Liao and Huang (2010).

4. **Optimization of Two-warehouse model**

The annual net profit per unit time is

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$$Z(T)=\begin{cases} Z_1(T), if\ 0<T\leq M \\ Z_2(T), if M<T\leq T_o \\ Z_3(T), if T_o<T \end{cases} \tag{23}$$

Also, at $T=T_o$ and $T_w=0$ the capacity of the warehouse is $W=\frac{\alpha}{\eta_o}\left(e^{\eta_o T_o}-1\right)$. $Z(T)$ is continuous function except at $T=T_o$.

We have, $Z_1^{'}(M)=Z_2^{'}(M)$

$$=\frac{1}{M^2}\left[\begin{array}{l} A+\frac{\alpha}{\eta_o^2}\left((1-\eta_o M)e^{\eta_o M}-1\right)\begin{pmatrix} h_o-(P-C)\eta_o \\ -PI_e\beta \end{pmatrix} \\ +PI_e\alpha\left[\frac{1}{2}\left(1+\frac{\beta}{\eta_o}\right)M^2-\frac{M}{\eta_o}\left(e^{\eta_o M}-1\right)\right] \end{array}\right] \tag{24}$$

$$Z_2^{'}(T_o)=\frac{1}{T_o^2}\left[\begin{array}{l} A+\frac{\alpha}{\eta_o^2}\begin{pmatrix} (1-\eta_o T_o)e^{\eta_o T_o} \\ -1 \end{pmatrix} \\ \qquad \begin{pmatrix} h_o-(P-C)\eta_o \\ -PI_e\beta \end{pmatrix} \end{array}\right]$$

$$+\frac{1}{T_o^2}\left[PI_e\alpha\begin{pmatrix} \frac{1}{2}\left(1+\frac{\beta}{\eta_o}\right)T_o^2 \\ +T_o e^{\eta_o T_o}(M-T_o) \\ -\frac{M}{\eta_o}\left(e^{\eta_o T_o}-1\right) \end{pmatrix}\right]$$

$$-\frac{1}{T_o^2}\left[\frac{CI_c\alpha}{\eta_o^2}\begin{pmatrix} (\eta_o T_o-1)e^{\eta_o(T_o-M)} \\ -\eta_o M+1 \end{pmatrix}\right] \tag{25}$$

and $Z_3^{'}(T_o)=\frac{1}{T_o^2}f_3(T_o)$. (26)

For simplicity, let

$$f_2(M)=\left[\begin{array}{l} A+\frac{\alpha}{\eta_o^2}\left((1-\eta_o M)e^{\eta_o M}-1\right)\begin{pmatrix} h_o \\ -(P-C)\eta_o \\ -PI_e\beta \end{pmatrix} \\ +PI_e\alpha\left[\frac{1}{2}\left(1+\frac{\beta}{\eta_o}\right)M^2-\frac{M}{\eta_o}\left(e^{\eta_o M}-1\right)\right] \end{array}\right]$$

And

$$f_2(T_o)=A+\frac{\alpha}{\eta_o^2}\left((1-\eta_o T_o)e^{\eta_o T_o}-1\right)\begin{pmatrix} h_o-(P-C)\eta_o \\ -PI_e\beta \end{pmatrix}$$

$$+PI_e\alpha\begin{pmatrix} \frac{1}{2}\left(1+\frac{\beta}{\eta_o}\right)T_o^2+T_o e^{\eta_o T_o}(M-T_o) \\ -\frac{M}{\eta_o}\left(e^{\eta_o T_o}-1\right) \end{pmatrix}$$

$$-\frac{CI_c\alpha}{\eta_o^2}\begin{pmatrix} (\eta_o T_o-1)e^{\eta_o(T_o-M)} \\ -\eta_o M+1 \end{pmatrix}$$

Since $Z_2(T)$ is concave on $[M,\infty), Z_2^{'}(T)$ is decreasing on $[M,\infty)$ and $f_2(M)>f_2(T_o)$.

Also, $f_2(T_o)<f_3(T_o)$ and

$f_2(M)>0$ if and only if $T_1^*>M$ (27)

$f_2(M)>0$ if and only if $T_2^*>M$ (28)

$f_2(T_o)>0$ if and only if $T_2^*>T_o$ (29)

$f_3(T_o)>0$ if and only if $T_3^*>T_o$ (30)

So we have following decision making Table 1 for optimum cycle time.

Table 1 Optimum Cycle time

	$f_2(M)$	$f_2(T_o)$	Optimal Cycle time (Whichever gives maximum profit)
$f_3(T_o)>0$	≤ 0	≤ 0	T_1^* or T_3^*
	>0	≤ 0	T_2^* or T_3^*

	$f_2(M)$	$f_2(T_o)$	Optimal Cycle time (Whichever gives maximum profit)
$f_3(T_o)\leq 0$	≤ 0	≤ 0	T_1^* or T_o
	>0	≤ 0	T_2^* or T_o

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	>0	>0	T_3^*

In next section, we discuss two examples to illustrate the proposed problem.

Example 1 Let $\alpha = 400$ units/year, $\beta = 2\%, h_o = \$0.2$ unit/year, $hr = \$0.5$ unit/year, $\theta_o = 2\%, \theta_r = 5\%, M = 0.1$ year, $P = \$20$ /unit/year, $C = \$5$ /unit/year, $I_C = 15\%, I_e = 12\%, W = 300$ units/year. Then $f_3(T_o) = 80.51 > 0$. The optimal solution is given in Table 2 for different values of ordering cost, A.

Table 2 Optimal solution for Example 1

A	T_o	$f_2(M)$	$f_2(T_o)$	T^*	Q^*	$Z(T^*)$
1	0.738 9	<0	<0	$T_1^* =$ 0.078 9	31.62	6070.7 7
15	0.738 9	>0	<0	$T_2^* =$ 0.314 0	126.4 3	6018.0 0
10 0	0.738 9	>0	>0	$T_3^* =$ 1.028 8	420.1 1	5858.0 9

The observations are

(a) Optimal cycle time and purchase quantity are very sensitive to ordering cost. The annual profit per unit time decreases with increase in A. It indicates that retailer should have larger order in a larger cycle time.

(b) For A=1 and A=15, the order quantity is lower than the storage capacity W. so the retailer should not go for rented warehouse. It suggests that lower ordering cost will facilitate the retailer to be with own warehouse.

Example 2 Let $D = 60$ units/year, $\beta = 2\%, h_o = \$4$ /unit/year, $h_r = \$5$ /unit/year, $\theta_o = 30\%, \theta_r = 35\%, M = 0.1$ year, $P = \$5$ /unit/year, $C = \$0.5$ /unit/year, $I_c = 14\%, I_e = 12\%, W = 15$ unit/year. Then $f_3(T_o) = -56.29 \le 0$, Using table 1, we have optimal solution in Table 3 for different values of A.

A	T_o	$f_2(M)$	$f_2(T_o)$	T^*	Q^*	$Z(T^*)$
1	0.207 3	<0	<0	0.094 6	6.72	298. 22
4	0.207 3	>0	>0	0.198 7	14.3 6	277. 75

Observations are similar to those stated for example 1.

Using data of example 1, we carry out sensitivity analysis to find out critical inventory parameters which forces the retailer to opt for rented warehouse. Fig.4, Fig.5and Fig. 3srespectively for optimum purchase quantity, total annual profit per unit time and optimum cycle time

7. Conclusions

This study aims to analyze the need of rented warehouse when own warehouse has limited storage capacity and demand is stock dependent. The items in warehouses are subject to different deterioration rate. It is observed that offer of delay payment by the supplier to the retailer encourages for a larger order. The decision policy is suggested to the retailer to use the rented warehouse to maximize profit. The numerical examples and sensitivity analysis will help the decision maker to take the favorable decision. The future study should consider time-dependent shortages, time-dependent deterioration of units etc.

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Fig. 1 $T_o \geq T$

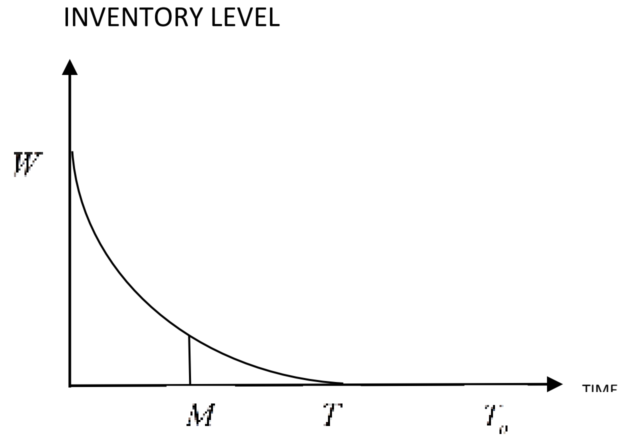
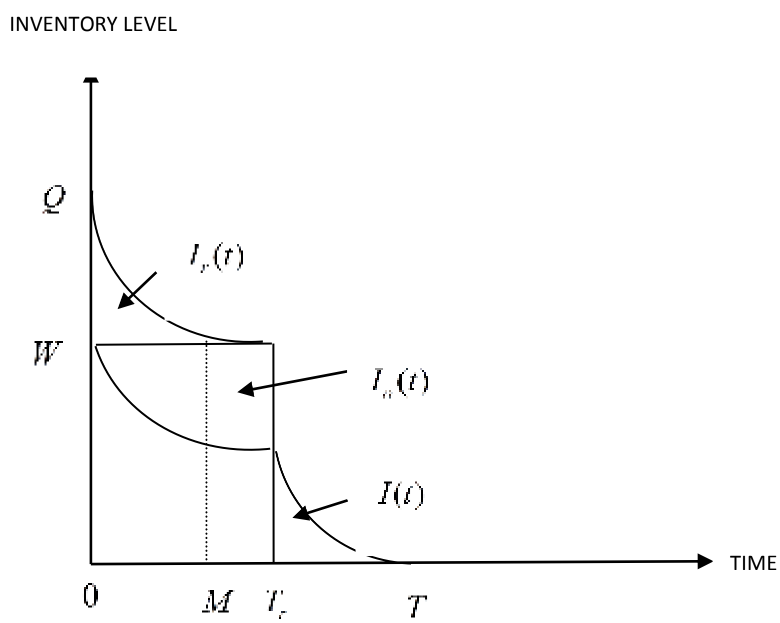


Fig. 2 $T_o < T$



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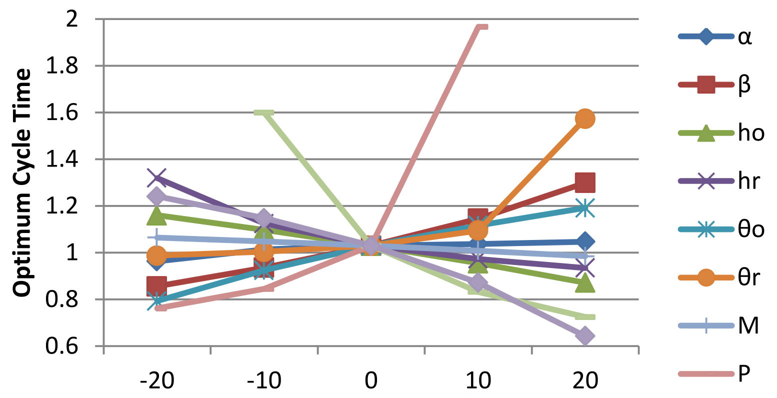

Fig. 3 Percentage Change in Inventory Parameters

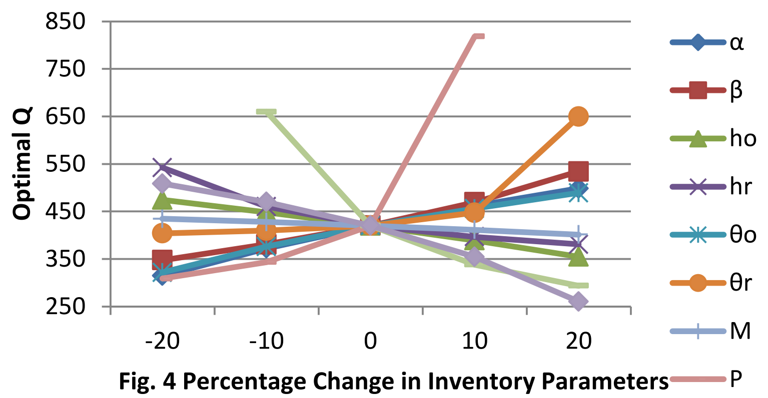

Fig. 4 Percentage Change in Inventory Parameters

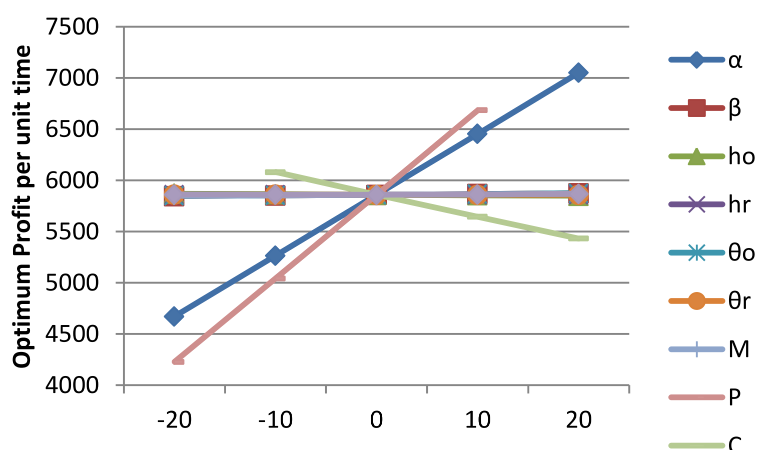

Fig. 5 Percentage Change in Inventory Parameters