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LOGISTIC FUNCTION AS A FORECASTING MODEL: IT'S APPLICATION TO BUSINESS AND ECONOMICS

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Abstract

Accurate forecasting is essential and of high importance for the business community and the economic world. In the attempt to understand the business and economic systems, mathematical models are utilized for its descriptive and predictive capabilities. Thus, the primary objective of the paper is to explore and apply logistic equation as a mathematical model in explaining and forecasting different areas in business and economics such as product diffusion and market acceptance, inflation rate of goods, purchasing power of peso, and employment and unemployment in the Philippines. Results show that logistic equation can be used to predict or describe some business and economic systems as well as describing the effects of the measures employed by the different sectors as to the problems associated with the areas of concern.

Keywords: mathematical models, business and economics, differential equations, logistic equations

1.0 INTRODUCTION

In the year 2008, the world had experienced one of the most tragic events in economic history – the infamous global economic crisis. Several causes have been pinpointed beginning with failures caused by misapplication of risk controls for bad debts, collateralization of debt insurance and fraud. The crisis rapidly developed and spread into a global economic shock, resulting in a number of European bank failures, declines in various stock indeces, and large reductions in the market value of equities and commodities. By the end of October a currency crisis developed, which led investors to transfer their capital resources into stronger currencies leaving many emergent economies to seek aid from the International Monetary Fund [13]. Due to this global financial crisis, many developing countries in Asia had experienced the most difficult economic challenges and the Philippines is not an exception to that.

To understand the global economic crisis and its possible effects, it is but necessary to look for a method which will give forecasts or overview of what might happen in the sector of business and economics. Forecasts are necessary for planning, decision-making, understanding and implementing prospective choices. In relation to this, economics has to use both quantitative and qualitative methods in explaining the economic situation of the market. Though in most instances, it uses the quantitative methods of mathematics.

Several well-known economists in the likes of Thomas Malthus, Evsey Domar and Pierre Verhaulst, utilized mathematical tools in trying to explain their most profound contributions to the field of economics usually as a model or a formal mathematical statement of a presumed relationship between two or more variables. Majority of the models developed express the relationships among the variables in terms of equations and system of equations. Among the known models are least square linear regression, exponential smoothing and moving averages.

Moreover, other models involve differential equations in the representation of the relationships among economic variables which concern changes over periods of time.

One of the many great results of ordinary differential equations is the derivation of the logistic equation $P(t) = \frac{MCe^{rt}}{1+Ce^{rt}}$. Several studies [3], [4], [5], [6], [7], [12] have revealed the usefulness of logistic equations in forecasting events in the field of physical, biological and social sciences.

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Gompertz curve (equation), a type of logistic equation showed significant results concerning market penetration of different products as stated in the study of Lackman [8]. In his paper, it should be noted that the econometric Gompertz equation had performed well in forecasting the sales of the relatively new specialty plastic in the market starting with sales data of the third quarter of 1986. Unfortunately not all forecast made by the model was that accurate as emphasized by Aravantos and Fallah [4]**.** Hence, in order to forecast telecommunications growth accurately, analogy and interpolation of observed trends from different countries, aside from careful selection of saturation level were made.

Because of its theoretical consistency with the micromolecular model of asphalt aging, logistic differential equation was used by Garrick [6] in representing changes in asphalt properties during pavement aging in the field. Its specific use was in the quantification of the rate of aging and the ultimate degree of change in asphalt properties due to aging.

To further prove is worth, logistic equations finds its use in the field of biological and social sciences as suggested by the studies of Bohner and Warth [5] and Kooi, Boer and Kooijiman [7]**.** Series of proofs for the Cushing-Henson were made by Bohner and Warth [5] to justify the conjecture that a periodic environment is deleterious for the population. The study also concentrated on the finding the optimal harvesting policy and optimum population level.

Kooi**,** Boer and Kooijiman [7] assumed that food chain models in the trophic level grow logistically. Bifurcation diagrams revealed that dynamic behaviours differ in large regions of the control parameters space due to dilution rate and concentration of resources. Zimm [12] devoted his time in the deductive derivation of a logistic equation for organization in a competitive economy. The construct is based on a system that consists of one or more organizations, each with its own cost, productivity, reinvestment parameters and each having its individual population of employees. It was revealed in his study that the model is chaotic but can be shifted to become deterministic in nature by adding a number of real world complications.

From the preceding discussions, it can be gleaned that logistic equation can be used to predict, forecast or describe a system which involves growth and decay. Such is in the case of some business and economic problems that will be discussed in this paper. However, majority of the studies mentioned focused on the application of logistic differential equations with the physical, natural and medical sciences while the present study dealt with the application of the logistic equation in business and economics, thus, the difference.

The researcher is also motivated to gain an in-depth understanding of the nation's economic condition and eventually contribute in the understanding of the situation. The study aimed to explore and apply the logistic equation as an alternative method in solving problems in business and economics.

This paper is deemed necessary to increase people's awareness on the potential use and applications of mathematics in the affairs of man in the world of business. Moreover, it may help people understand and form association on the role and principle of mathematics in other fields of endeavor.

2.0 METHODOLOGY

The research paper is descriptive-expository in nature which was focused on the areas of Philippine business and economics that fit the logistic curve/function. Variation of the logistic equation is also discussed in the paper.

In order to clarify some concepts regarding the research study, experts were consulted. For the purpose of verifying the model, documentary analysis of data provided by the Philippine National Statistics Office and other statistical sites was used. This includes product diffusion and market acceptance, inflation rate of goods, purchasing power of peso, and employment and unemployment.

Since mathematical models approximate only the system or phenomena, the model presented in this study may have its limitation such that results may vary greatly from the actual data and information. All approximated values were derived using the specific solution for the initial value problems. Furthermore, unless otherwise specified, all known and unknown factors were held constant in the forecasts of the business and economic events.

3.0 RESULTS AND DISCUSSIONS

The subsequent sections discuss the concept of logistic equation and its applications to business and economics.

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3.1 The Logistic Function

As illustrated by the Malthusian growth law, population is exponentially increasing over time. However, this law does not give an accurate model for growth of a population over a long period of time [6], [10]**.** Growth usually occurs with bounds or limitations. When the system reaches saturation point, certain limitations will arise; hence, restricting its growth. Such limitations include limited space and resources, famine and diseases; hence the population as predicted by the Malthusian law would soon reach its maximum sustainable population and would not increase forever.

The logistic equation, also known as the Verhulst model, named after Pierre Verhulst, is an example of bounded growth which is limited by saturation with respect to time. It is assumed that the growth rate of the population is proportional to number of individuals present, *P(t),* but is constrained by how close the number *P(t)* is to the maximum population, *M* [5]. The differential equation that describes this is

$$\frac{dP}{dt} = \frac{rP}{M}\left(M - P\right) \qquad (1)$$

where r is the rate of maximum population growth or the Malthusian parameter. The solution to this equation is the logistic equation (function) derived by simple separation of variables and algebraic manipulations.

3.2 Logistic Equation and the Bernoulli's Equation

Another way of deriving the logistic function is by using the method developed by Jacob Bernoulli.

Theorem 3.1. The differential equation $\frac{dP}{dt} = \frac{rP}{M}\left(M - P\right)$ is a Bernoulli's equation where n = 2.

Proof. If the differential equation *(1)* is expressed in the form $\frac{dP}{dt} - rP = -\frac{rP^2}{M}$, it has been reduced to a Bernoulli's equation where n = 2. ■

The general solution to this equation is the logistic equation. The particular solution is given by the equation

$$P(t) = \frac{MP_0}{(M - P_0)e^{-rt} + P_0} \qquad (2)$$

where M is the maximum capacity for growth, P_0 is the initial value at $t = 0$ ($P_0 \neq 0$), r is the rate of growth and t is the time.

The parameters of the specific solution, which are determined using data points of equal time interval $\{(t_0, P_0), (t_1, P_1), (t_2, P_2)\}$ are given by the following equations.

$$r = \frac{1}{t_1} ln\left|\frac{P_2\left(P_1 - P_0\right)}{P_0\left(P_2 - P_1\right)}\right| \qquad (3)$$

$$M = \frac{P_1\left[P_0P_1 + P_1P_2 - 2P_0P_2\right]}{P_1^2 - P_0P_2} \qquad (4)$$

where $t_1 \neq 0$; $P_0, P_1, P_2 \neq 0$.

3.5 Areas in Business and Economics that Fit the Logistic Function

The succeeding subsections discuss the applications of logistic function to areas of business and economics.

3.5.1 Product Diffusion and Market Acceptance

Product diffusion and market acceptance of new products or services and innovations are perhaps few of the major concerns of the businessmen and producers. Not only will these determine their advantages or disadvantages over competitors but also the plans that could be implemented as needed. Moreover, the effectiveness of marketing strategies used by business enterprises can be measured by means of how fast the market will patronize their product or how fast the product will be dispatched.

Owing to higher prices of new products at the beginning, only the people belonging to high income group has the capacity to buy the goods and only by the time that these products become less expensive that they were diffused to lower income groups. However, as soon as the market had been saturated by the product, the rate of diffusion will decline asymptotically for it cannot grow above the market population. This inference about product diffusion clearly shows the characteristic of the logistic equation.

To verify the assumption that product diffusion and market acceptance of new products the actual data for the number of Philippine mobile cellular telephone subscribers from year 2000 to 2009 are be used.

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Table 1 shows the actual number of mobile telephone subscribers in the Philippines (2000 – 2009) and approximated/predicted number of subscribers at different time intervals. At time interval of two years (t_1 = 2), the value of r is 0.699318183365703 and M is 27 093 225. At time interval of three years (t_1 = 3), the value of r is 0.663953418210177 and M is 46 786 970. At time interval of four years (t_1 = 4), the value of r is 0.564592196126985 and M is 72 036 179. Notice that the rate of growth r, is inversely proportional with the maximum capacity for growth, M.

Table 1. The Actual and Approximate Number of Mobile Cellular Telephone Subscribers in the Philippines

t	Year	Actual*	Approximate $t_{1=2}$	%Error	Approximate $t_{1=3}$	%Error	Approximate $t_{1=4}$	%Error
0	2000	1959000	1959000	0.0	1959000	0.0	1959000	0.0
1	2001	1959000	3673359	87.5	3660812	86.9	3375701	72.3
2	2002	6500000	6500000	0.0	6622605	1.9	5733107	- 11.8
3	2003	11350000	10524302	- 7.3	11350000	0.0	9508803	- 16.2
4	2004	15201000	15201000	0.0	17944288	18.0	15201000	0.0
5	2005	15201000	19508940	28.3	25601889	68.4	23044833	51.6
6	2006	32810000	22706663	- 30.8	32810000	0.0	32613591	- 0.6
7	2007	32810000	24720147	- 24.7	38371724	17.0	42693130	30.1
8	2008	51795000	25859628	- 50.1	42040487	- 18.8	51795000	0.0
9	2009	51795000	26465850	- 48.9	44216921	-14.6	58939649	13.8

*Source: *www.indexmundi.com*

It can be gleaned from Table 1 that the approximations made at time intervals of two (t_1 = 2) and three years (t_1 = 3) agrees quite well with the observed number of mobile telephone subscribers for the first five years from 2000 to 2004, but the approximation thereafter are too small. Mathematically speaking, this could be explained by the limited capacity for saturation for these time intervals. This is not surprising for the model fails to take into account such factors as promotional activities, cellular phone models, and surplus of cheaper units.

Compared with the first two approximations, the values at time interval of four years (t_1 = 4) gives the closest approximated values to the actual number of mobile cellular telephone subscribers as evident by the lesser margin of errors. Unlike the time intervals t_1 = 2 and t_1 = 3, the time interval t_1 = 4 gives provision for further growth of the number of mobile phone subscribers.

Figure 2 shows the logistic curves with the initial value P_0 = 1 959 000 at different values of r. It is observed that as the value of r decreases, the logistic curve becomes steeper and steeper. From this figure, it is observed that at time intervals t_1 = 2 and t_1 = 3, growth gradually slows down and approaches their maximum capacity for growth immediately after the ninth year. But the logistic curve at r = 0.564592196126985 (t_1 = 4), being the steepest of the curves, continues to follow the actual growth of the mobile subscribers.

The choice of values for P_0, P_1, P_2 is very important in computing the values of r and M for it will determine the steepness of the curve; hence, the possibility of a more closer approximation.

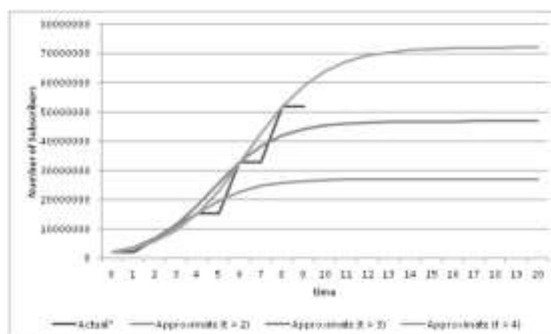

Figure 1. Graph showing the actual and approximate number of mobile cellular telephone subscribers in the Philippines

3.5.2 Inflation Rate and Prices of Goods

If people can correctly anticipate the rate of inflation and adjust prices as well as incomes, then the costs of inflation would be relatively small.

Suppose that inflation rate the follow the logistic curve. Then by equations (3) and (4) at an interval of every quarter (t_1 = 3) from December

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2007 – December 2009 data on inflation rate, r = 0.377409633155698 and M = 8.03.

Table 2 compares the values predicted in by the logistic function with the actual values of the inflation rates. The predicted values are closer with the actual inflation rates from December 2007 to December 2009 but the predicted values are higher for the months of January 2009 to January 2010. This may be attributed to the fact that the model did not consider other factors which could affect the prices of goods as well as the demand and supply.

Table 2. Philippine Inflation Rates (Dec 2007- Dec 2009)

t	Month - Year	Actual*	Approx.	%Error
0	Dec-07	2.6	2.6	0.0
1	Jan-08	3.4	3.3	-2.9
2	Feb-08	4.0	4.1	1.3
3	Mar-08	4.8	4.8	0.0
4	Apr-08	5.9	5.5	-6.9
5	May-08	6.2	6.1	-1.6
6	Jun-08	6.6	6.6	0.0
7	Jul-08	6.3	7.0	11.0
8	Augt-08	7.0	7.3	4.1
9	Sep-08	7.5	7.5	0.1
10	Oct-08	7.8	7.7	-1.7
11	Nov-08	7.9	7.8	-1.6
12	Dec-08	7.3	7.9	7.6
13	Jan-09	6.9	7.9	14.6
14	Feb-09	6.4	7.9	24.2
15	Mar-09	5.6	8.0	42.4
16	Apr-09	5.0	8.0	59.9
17	May-09	4.4	8.0	81.9
18	Jun-09	3.9	8.0	105.5
19	Jul-09	3.6	8.0	122.8
20	Aug-09	2.9	8.0	176.7
21	Sep-09	2.8	8.0	186.7
22	Oct-09	2.7	8.0	197.4
23	Nov-09	2.7	8.0	197.4
24	Dec-09	3.2	8.0	151.0
25	Jan-10	3.0	8.0	167.7

*Source: *www.nso.gov.ph*

This only suggests that the Philippine government is successful in the implementation of economic measures in decreasing high inflation rates.

The prices of goods rose quickly from February 2008 – November 2008 and then gradually decrease after. This simply suggests that during those months, prices of goods increase rapidly than the previous months and then decrease after. The decrease in inflation does not mean that prices decrease in values; rather it means that prices increase in slower pace, as suggested by the deviations seen after November 2008, when prices of goods started to increase slower than the previous price increase.

Figure 2 shows the graph of the logistic curve and the actual values. Observe that the logistic curve approximate the values quite close to the original values as indicated by the overlapping curves.

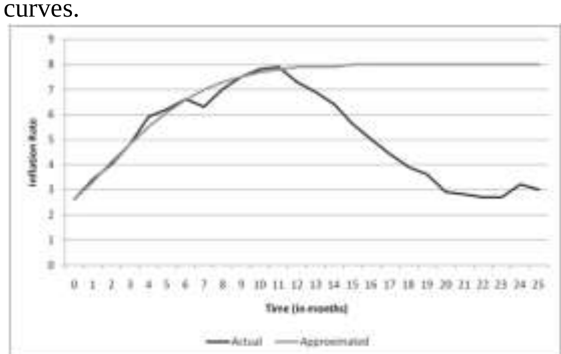

Figure 2. Graph showing the actual and the approximated inflation rates by months.

3.5.3 Purchasing Power of Peso

Another factor used to determine economic growth is the purchasing power of the currency. If a country has a relatively high inflation rate then the value of its currency will decline**.** Hence, firms will constantly raise prices in the attempt to cover their increasing productions costs. As a consequence, the workers will demand for huge pay increase in the attempt to stay ahead of the increasing costs of goods. Consumers will no longer want to save money but will spend it as quickly as possible before its value falls any further.

Suppose that the purchasing power of peso follow the logistic curve. Then by equations (3) and (4), at an interval of every six months from December 2007 – October 2009 data on purchasing

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power of peso, r = -0.122617799163124 and M = 0.729452054794513. Time intervals of six months have been used due to closeness of values on other time intervals.

It can be gleaned from Table 3 that the approximation of the value of Purchasing Power of Peso (PPP) are quite closer with the actual values, with margin of error ranging from – 3.5 to 2.1. It is very obvious that the PPP follows the logistic function/curve.

Table 3. Purchasing Power of Peso (Dec 2007–Oct 2009)

t	Month - Year	Actual	Approx	%Error
0	Dec-07	0.72	0.7200	0.00
1	Jan-08	0.71	0.7188	1.24
2	Feb-08	0.72	0.7174	-0.36
3	Mar-08	0.72	0.7159	-0.57
4	Apr-08	0.71	0.7141	0.58
5	May-08	0.71	0.7122	0.31
6	Jun-08	0.71	0.7100	0.00
7	Jul-08	0.70	0.7075	1.08
8	Aug-08	0.70	0.7048	0.68
9	Sep-08	0.70	0.7017	0.24
10	Oct-08	0.70	0.6982	-0.26
11	Nov-08	0.69	0.6943	0.63
12	Dec-08	0.69	0.6900	0.00
13	Jan-09	0.69	0.6852	-0.70
14	Feb-09	0.68	0.6798	-0.03
15	Mar-09	0.67	0.6738	0.57
16	Apr-09	0.66	0.6672	1.08
17	May-09	0.65	0.6598	1.51
18	Jun-09	0.64	0.6517	1.83
19	Jul-09	0.63	0.6428	2.02
20	Aug-09	0.62	0.6329	2.09
21	Sept-09	0.63	0.6222	-1.24
22	Oct-09	0.63	0.6105	-3.10

*Source: *www.nso.gov.ph*

Furthermore, based on the table, the purchasing power of Philippine peso continuously decreases its value. Moreover, it was approximated by the logistic curve/function that by the end of the first quarter of 2010, the purchasing power of peso will be at 0.53 to 0.54.

Figure 3 shows how close the logistic curve with the curve of the actual values for the Purchasing Power of Peso. Notice that the logistic curve intersects the curve of the actual value of PPP.

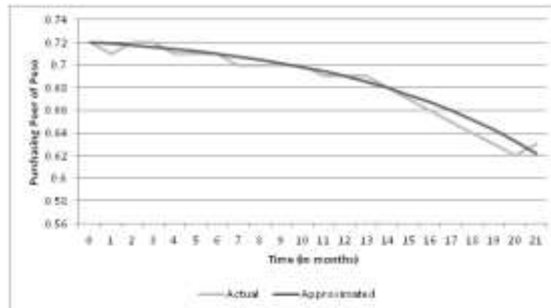

Figure 3. Graph showing the actual and the approximated value of the Purchasing Power of Philippine peso

3.5.4 Employment and Unemployment

The employment statistics in the Philippines shows that there is the incidence of low employment rate in the country. It had always been the goal of the government to meet full employment, but for some known and unknown reasons, it is difficult to provide enough employment opportunities to the people. Thus, it is vital to have some measure of the growth of employment not only to determine how it is affected by the global recession but also to determine how effective are the measures taken to address the problem of unemployment.

Suppose the number of total labor force and unemployed persons follows the logistic curve.

Table 4 and Table 5 show the total labor force and number of unemployed workers in the Philippines. With the given initial condition and at an interval of every three years (t_1 = 3), the growth rate is 1.24085432462072 and will have a maximum capacity for growth equal to 36 811 986 for labor force. At an interval of every three years (t_1 = 3), the growth rate is -0.300073495363569 and will have a maximum capacity for growth equal to 3 625 345 for unemployment.

Based on these tables, it can be concluded that the actual data fit the logistic function. Hence, it can be a good estimator of the effectiveness of the measures employed by the government in relation to problems of unemployment in the country.

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Table 4. Total Labor Force

t	Year	Actual*	Approx	% Error
0	2003	33,700,000	33,700,000	0.0
1	2004	34,560,000	35,854,666	3.7
2	2005	35,860,000	36,529,976	1.9
3	2006	36,730,000	36,730,000	0.0
4	2007	35,790,000	36,788,243	2.8
5	2008	36,220,000	36,805,118	1.6
6	2009	36,810,000	36,810,000	0.0

*Source: *www.nso.gov.ph*

Table 5. Total Number of Unemployed Persons

t	Year	Actual	Approx.	%Error
0	2003	3,437,400	3,437,400	0.0
1	2004	3,939,840	2,904,478	-26.3
2	2005	4,195,620	2,711,397	-35.4
3	2006	3,195,510	2,651,741	-17.0
4	2007	2,827,410	2,634,156	-6.8
5	2008	2,644,060	2,629,043	-0.6
6	2009	2,723,940	2,627,562	-3.5

*Source: *www.nso.gov.ph*

3.5.5 Inflation Rate and Purchasing Power of Peso

As pointed out earlier, the inflation rate has a great influence with the purchasing power of peso. Hence, it will be appropriate if inflation will be considered as a factor for approximating the purchasing power of peso.

Suppose the rate of change in the purchasing power of peso is a function of inflation rate. Then by using the initial value problem $\frac{dP}{dt} = rP\left[a - bP\right]$ where $a - bP$ is the inflation rate function derived using the least square linear regression analysis.

From the linear regression formula, the values of a and b are - 6.541270846527 and 17.601857715854, respectively. Hence, the value of r is 0.00086132733615984 at $t_1 = 5$ (for every 5 years).

Table 6 summarizes and compares the actual values from the approximated values on purchasing power of peso. It can be gleaned from the table that the approximate values of the purchasing power of peso do not vary greatly from the actual for the first few months. However, notable deviations can be seen from the approximated values since the value of e^{-art} increases, at $t \rightarrow \infty$. Hence, a gradual decrease in the approximated value. It should also be noted that the choice of time interval yields different approximations.

Table 6. Inflation Rate and Purchasing Power of Peso (Dec 2007–Oct 2009)

t	Month – Year	IR	PPP	Approx	% Error
0	Dec-07	2.6	0.72	0.72	0.00
1	Jan-08	3.4	0.71	0.71	-0.25
2	Feb-08	4.0	0.72	0.70	-3.22
3	Mar-08	4.8	0.72	0.69	-4.77
4	Apr-08	5.9	0.71	0.67	-4.95
5	May-08	6.2	0.71	0.66	-6.44
6	Jun-08	6.6	0.71	0.65	-7.89
7	Jul-08	6.3	0.70	0.64	-8.01
8	Aug-08	7.0	0.70	0.63	-9.41
9	Sep-08	7.5	0.70	0.62	-10.78
10	Oct-08	7.8	0.70	0.62	-12.11
11	Nov-08	7.9	0.69	0.61	-12.15
12	Dec-08	7.3	0.69	0.60	-13.44
13	Jan-09	6.9	0.69	0.59	-14.69
14	Feb-09	6.4	0.68	0.58	-14.69
15	Mar-09	5.6	0.67	0.57	-14.65
16	Apr-09	5.0	0.66	0.56	-14.58
17	May-09	5.0	0.65	0.56	-14.48
18	Jun-09	4.4	0.64	0.55	-14.35
19	Jul-09	3.9	0.63	0.54	-14.19
20	Aug-09	3.6	0.62	0.53	-14.00
21	Sep-09	2.9	0.63	0.53	-16.52

*Source: *www.nso.gov.ph*

Furthermore, it should also be noted that the large %error signifies the effectiveness of the government's measure to slow down inflation rates and to increase the purchasing power of peso.

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4.0 CONSLUSIONS AND DIRECTIONS FOR FUTURE USE

Aside from the known forecasting methods and techniques, the logistic function can also be used as a good forecasting model. It can significantly determine at certain degree of accuracy how effective are the measures implemented to a certain concern in business and economics. Based on the foregoing findings, the logistic functions give close approximations of the actual values. But, appropriate choice of time interval should be considered as it affects greatly the approximation of the predicted values from the logistic equation.

The different business and economic problems that follow a logistic curve are product diffusion or market acceptance, inflation rate of goods, purchasing power of peso, and employment and unemployment.

Another important area where logistic function can be used is in the area of climate and disaster management, and sustainability problem.

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