SORET AND MAGNETIC EFFECTS ON THE ONSET OF CONVECTION IN A SHALLOW HORIZONTAL POROUS LAYER WITH THERMAL RADIATION

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ABSTRACT

Convective flow in a shallow porous layer is studied. The horizontal channel is subjected to magnetic field and flux of heat and mass. Using the Darcy model with the assumption that the fluid is Newtonian and radiating, the governing equations were prescribed. For given values of the governing parameters, we obtained our numerical solutions using mathematica. Our results shows that a change in the sign of the stream function changes the direction of the flow from clockwise to anticlockwise or vice versa without changing the characteristics of the flow. The effect of Soret, magnetic field, radiation parameter, thermal Rayleigh number on the flow and on the heat and mass transfer depends on whether the solutal and thermal buoyancy forces are cooperative or not (both cases are discussed) but a common feature exist in which increases in electromagnetic force field decreases the flow intensity. For the heat driven flow limit, the flow intensity for the clockwise and counterclockwise solutions are equal and the stream function and heat transfer rate are not dependent on Soret at each thermal Rayleigh value.

Key Words: Soret, magnetic field, double diffusive convection, thermal Rayleigh number, porous medium

1. INTRODUCTION

Flows induced by the buoyancy forces resulting from the imposition of both thermal and solutal boundary conditions on the layer are of interest principally because of its wide range of applications relevant to man. For example, geographical systems, solidification of binary mixtures, solute intrusion in sediments in coastal areas, specie transport through biological membranes, electrochemistry, engineering applications involving sudden heating or cooling and many others.

Early studies on double diffusive natural convection in porous media focused on the onset of convective instability in a horizontal porous layer. On the basis of linear stability analysis theory, Malshetty (1993), Nield (1986), Poulikakos (1986), and Bahloul et al. (2003) investigated the onset of double diffusive convection. Rudraiah et al. (1982), and Israel-Cookey and Amos (2014) also used linear stability analysis to study instabilities and diffusive regimes for double diffusive convection.

The problem of double diffusive convection and cross-diffusion in a Maxwell fluid in a horizontal porous layer was studied by Award et al. (2010) using the modified Darcy-Brinkman model and analytical expressions of the critical Darcy-Rayleigh number for the onset of stationary and oscillatory convection were derived. Other investigations on the onset of stationary and oscillatory convections include Amahmid et al. (1999) and Liu et al. (2008).

Experimental studies on thermodiffusion phenomena proved the existence of mixtures in which the Soret coefficient is large enough to affect the flow and heat and mass transfer in these mixtures considerably. Platen and Costeseque (2004) determined experimentally the effect of Soret coefficient in pure fluid and porous medium and concluded that this coefficient is the same for both media. Benano-Melly et al. (2001) studied numerically and experimentally the problem of thermo-diffusion in an initially homogenous mixture submitted to a horizontal thermal gradient. Their numerical results showed that,
depending on Soret coefficient value, multiple convection-roll flow patterns could develop in the presence of counter-acting thermal and solutal buoyancy forces. In their study of double diffusive natural convection, Mansour et al. (2006), Amos and Israel-Cooke (2015) concluded that Soret coefficient considerably affects the heat and mass transfer depending on the flow structure and the sign of the Soret parameter. Er-Raki et al. (2006) studied the effect of Soret on boundary layer flows induced in a vertical porous layer subjected to horizontal heat and mass fluxes. They showed that, depending on the sign of the buoyancy ratio, the boundary layer thickness can increase or decrease with the Soret coefficient.

An investigation of the electromagnetic stabilization of convection in a rather simple numerical model leads to several qualitative conclusions which will be valid also for more complicated configurations and therefore relevant to the technological processes mentioned above. Theoretical and numerical studies of thermal convection under the action of constant magnetic field were conducted for various geometries by Oreper and Szekely (1983), Di Piazza and Ciofalo (2002), Kim et al. (1988), and Bojarevias (1995). Also, the effect of magnetic field on double diffusive convection in a binary fluid saturating a horizontal layer was studied by Rakoto-Ramambason and Vasseur (2007). According to reasonable simplifications, Siegal and Howell (1972), and Israel-Cooke and Omubo-Peppe (2007) considered radiative heat flux in the energy equation to describe convective flow.

In all these investigations reported, the simultaneous effect of Soret coefficient and magnetic field on thermostolutal convection in a shallow porous medium taking into consideration the presence of radiative heat flux has not been considered. That is the main purpose of this present study.

2. MATHEMATICAL FORMULATION

The flow configuration consists of a two dimensional shallow enclosure of height, \( H \) and length, \( L \), permeability, \( K \) and filled with electrically conducting binary fluid (see figure 1).

The binary fluid that saturates the porous layer is modeled as Boussinesq incompressible fluid and the state equation is

\[
\rho = \rho_0 \left[ 1 - \beta_T (T' - T_0) - \beta_C (C' - C_0) \right] \quad (1)
\]

where \( \beta_T \) and \( \beta_C \) are thermal expansion coefficient and concentration expansion coefficient respectively, subscript zero refers to reference state. The horizontal walls are subject to uniform fluxes of heat, \( q' \) and mass, \( j' \) per unit area. The effects due to viscous dissipation and porous medium inertial forces are negligible. A magnetic field of intensity \( \vec{B}_0 \) is applied perpendicular to the channel. Using the Darcy model and taking into account the Soret effect, with the assumption that the fluid is Newtonian and radiating, the dimensional governing equation expressing continuity, momentum, energy and specie concentration are

\[
\vec{V}' \cdot \nabla' = 0 \quad (2)
\]

\[
\frac{\mu}{\kappa} \nabla' = -\nabla' P' + \rho \vec{g}' + j'_e \times \vec{B}_0 \quad (3)
\]

\[
(\rho c)_p \frac{\partial T'}{\partial t'} + \left( \rho c \right)_f \left( \vec{V}' \cdot \nabla' T' \right) = \kappa \nabla' T' - \nabla' \cdot \vec{q}' \quad (4)
\]

\[
\vec{q}' = \nabla' \cdot \vec{V}' \nabla' C' = D \nabla' C' + D^* \nabla' T' \quad (5)
\]

where \( \vec{g} \) is the gravitational acceleration, \( \epsilon' \) is the porosity of the porous medium, \( (\rho c)_p \) and \( (\rho c)_f \) are the heat capacity of the porous medium and the fluid respectively, \( \kappa \) is the thermal conductivity, \( \vec{V}' \) is the Darcy velocity, \( q'_f \) is the radiative flux, \( D \) and \( D^* \) are mass diffusivity of species and thermo-diffusion coefficient respectively, \( \rho \) is the pressure.
\( \mathbf{j}_c \times \mathbf{B}_0 \) is electromagnetic force which consists of the current density \( \mathbf{j}_c \) and the magnetic field \( \mathbf{B}_0 \), \( T' \) and \( C' \) are temperature and solutal mass concentration respectively.

Following Amos and Israel-Cookey (2015), equations (3) and (4) reduce to

\[
\nabla^2 \psi' = - \frac{\beta_T K}{\nu} \left( \frac{\partial T'}{\partial x'} + \frac{\beta_c}{\beta_T} \frac{\partial C'}{\partial x'} \right) - \frac{\beta_T^2 K}{\mu} \psi'- \frac{2H}{v} \partial^2 \psi',
\]

\[
\left( \frac{\partial x}{\partial x'} \right)_{p} \frac{\partial T'}{\partial x'} + \left( \frac{\partial x}{\partial y'} \right)_{f} \left( \nabla^2 \nabla T' \right) = \kappa \nabla^2 T' + \frac{16 \sigma^* T_0^2}{3 \delta} \nabla^2 T',
\]

(7)

where \( \psi' \) is the stream function and \( u = \frac{\partial \psi'}{\partial y'} \),

\( v = - \frac{\partial \psi'}{\partial x'} \) such that equation (2) is satisfied. \( K \) is the permeability of the porous medium, \( \sigma \) is electrical conductivity, \( \sigma^* \) is Stephan Boltzmann constant, \( \delta \) is the mean absorption coefficient. Equations (5), (6) and (7) and are to be solved subject to the boundary conditions:

\[
\psi' = 0, \quad \frac{\partial T'}{\partial x'} = 0, \quad \frac{\partial C'}{\partial x'} = 0 \quad \text{at} \quad x' = \pm \frac{L}{2}
\]

\[
\psi' = 0, \quad \frac{\partial T'}{\partial y'} = - \frac{q'}{k}, \quad \frac{\partial C'}{\partial y'} = - \frac{j}{D} \frac{D^* C_0 q'}{D} \quad \text{at} \quad y' = \pm \frac{H}{2}
\]

(8)

(9)

We introduce the following non dimensionless variables (primed quantities are dimensional)

\[
(x, y) = \left( \frac{x', y'}{H} \right), \quad (u, v) = \left( \frac{u', v'}{H} \right), \quad \alpha
\]

\[
t = \frac{t' \gamma}{\phi H^2}, \quad \varepsilon = \frac{\varepsilon'}{\phi}, \quad \theta = \frac{T' - T_0}{\Delta T'}
\]

\[
(\nu, R_T, C, \Delta C') = \frac{\psi'}{\psi}, \quad \Delta T' = \frac{q' H'}{\kappa}, \quad \Delta C' = \frac{j H'}{D}, \quad Le = \frac{\alpha}{\nu D} \quad (10)
\]

\[
R_T = \frac{\beta_T K q' H'^2}{\kappa \nu \alpha}, \quad R_C = \frac{\beta_C K j H'^2}{\nu D^2}.
\]

\[
v = \frac{\mu}{\rho_0}, \quad \text{where} \quad \phi = \frac{(\rho x)^p}{(\rho x)_f} \quad \text{is the heat capacity ratio.}
\]

Consequently we have the following governing equations based on equations (5), (6), (7), (8) and (9)

\[
\left( 1 + M^2 \right) \nabla^2 \psi' = \left( R_T \frac{\partial \theta}{\partial x} + R_C \frac{\partial C}{\partial x} \right) \quad (11)
\]

\[
\frac{\partial \theta}{\partial t} + \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \left( 1 + R^2 \right) \nabla^2 \theta \quad (12)
\]

\[
\varepsilon \left( \frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = \frac{1}{Le} \left[ \nabla^2 C + S \nabla^2 \theta \right] \right. \quad (13)
\]

subject to

\[
\psi' = 0, \quad \frac{\partial \theta}{\partial x} = 0, \quad \frac{\partial C}{\partial x} = 0 \quad \text{for} \quad x = \frac{A_R}{2}
\]

\[
\psi' = 0, \quad \frac{\partial \theta}{\partial x} = -1, \quad \frac{\partial C}{\partial y} = -1 + S \quad \text{for} \quad x = \frac{1}{2}
\]

(14)

(15)

\( R_T \) is the thermal Rayleigh number, \( R_C \) is the solutal Rayleigh number, \( A_R = \frac{L}{H} \) is the cavity aspect ratio, \( Le \) is the Lewis number, \( S \) is the Soret parameter, \( R \) is the radiation parameter and \( M \) is the magnetic parameter. The buoyancy is given as

\[
N = \frac{\beta_c}{B_T} \frac{\Delta C'}{\Delta T'} \quad (16)
\]

and is such that

\[
R_C = R_T \ N Le \quad (17)
\]
Hence we express the intensity of the thermal and solutal buoyancy forces in terms of \( R_T \) and \( R_C \).

The Nusselt number, \( Nu \), and the Sherwood number, \( Sh \), which expresses the heat and solute transport respectively, are

\[
Nu = \frac{1}{[\theta(0, -\frac{1}{2}) - \theta(0, \frac{1}{2})]}
\]

and

\[
Sh = \frac{1}{[C(0, -\frac{1}{2}) - \theta(0, \frac{1}{2})]}
\]

(18)

3. METHOD OF SOLUTION

In the limit of shallow cavity \((A_T \gg 1)\) we find approximate solutions (Bahloul et al. 2003) expressed in the parallel flow form:

\[
\psi(x, y) = \psi(y)
\]

(19)

\[
\theta(x, y) = D_\theta x + E_\theta(y)
\]

(20)

\[
C(x, y) = D_C x + E_C(y)
\]

(21)

We apply equations (19)-(21) on equations (11)-(15) and solving the resulting equations yields

\[
\psi = \psi_0 \left(1 - 4y^2\right)
\]

(22)

\[
\theta(x, y) = D_\theta x + (S - 1)y + \frac{\psi_0 D_\theta}{3(1 + R^2)} (3y - 4y^3)
\]

(23)

\[
C(x, y) = D_C x + (S - 1)y + \frac{\psi_0}{3} \left(Le D_C - \frac{SD_\theta}{1 + R^2}\right) (3y - 4y^3)
\]

(24)

where

\[
\psi_0 = \frac{R_T}{8(1 + M^2)} \left(D_\theta + ND_C\right)
\]

(25)

The equation for \( D_\theta \) and \( D_C \) can be established by imposing zero heat and mass transfer across any transversal section. That is

\[
\int_{-1/2}^{1/2} \left( u\theta - (1 + R^2) \frac{\partial\theta}{\partial x} \right) dy = 0 \quad (26)
\]

and

\[
\int_{-1/2}^{1/2} \left[u C - \frac{1}{Le} \left(\frac{\partial C}{\partial x} + S \frac{\partial\theta}{\partial x}\right)\right] dy = 0 \quad (27)
\]

The solutions to (26) and (27) establishes that

\[
D_\theta = \frac{4b(1 + R^2)\psi_0}{3[2b(1 + R^2)^2 + \psi_0^2]} \quad (28)
\]

\[
D_C = \frac{[Le \psi_0^2 - 2b(1 + R^2)]SD_\theta - 4bLe \psi_0(S - 1)(1 + R^2)}{3(1 + R^2) \left[2b + Le^2 \psi_0^2\right]} \quad (29)
\]

where \( b = \frac{15}{16} \)

From this statement, we can deduce expression for the heat and mass transfer respectively as

\[
Nu = \frac{18b(1 + R^2)^2 + 9\psi_0^2}{18b (1 + R^2)^2 + (9 - 8b)\psi_0^2} \quad (30)
\]

\[
Sh = \frac{3(1 + R^2)}{3(S - 1)(1 + R^2) + 2\psi_0 [Le D_C (1 + R^2) - SD_\theta]} \quad (31)
\]

Setting \( \psi_0 = 0 \) in equations (23) and (24), a purely diffusive state is obtained. That is

\[
\theta(x, y) = -y \quad (32)
\]

\[
C(x, y) = (S - 1)y \quad (33)
\]
Thus for the purely diffusive state, the heat and mass transfer is

\[ Nu = 1 \]  \hspace{1cm} (34)  

and

\[ Sh = \frac{1}{1 - S} \]  \hspace{1cm} (35)  

Substituting equations (28) and (29) into equation (25) yields.

\[
\psi_0 \left[ Le^2 \psi_0^4 - 2b Le d_1 \psi_0^2 - b^2 d_2 \right] = 0  \hspace{1cm} (36)
\]

where

\[
d_1 = \frac{1}{12} \frac{R_T}{(1 + M^2)} \left[ Le (1 + R_T^2) + NS - N(S - 1) \right]
\]

\[
d_2 = \frac{1}{3} \frac{R_T}{(1 + M^2)} \left[ -NS - NLe (S - 1)(1 + R_T^2) \right] - \frac{2}{3} (1 + R_T^2)^2
\]

4. RESULTS

The evolution of the flow intensity \[ |\psi_0| \], the heat transfer \[ Nu \], and the mass transfer, \[ Sh \], depends on the values attributed to the governing parameters. For given values of \[ R_T, Le, N, R, S \], equation (36), is solved numerically using the software ‘mathematica’ Wolfram, (1991). Five different solutions are predicted by this model. Apart from the rest state solution, our observation of the results shows that a change in the sign of \[ \psi_0 \], changes the direction of the flow from clockwise to counterclockwise or vice versa without changing the flow intensity or the characteristics of the heat and mass transfer. Points in the table with ‘\text{--}’ indicates entry which otherwise would have been a complex value.

4.1. Solutal Dominated Aiding Flow

For the case of cooperative solutal and thermal buoyancy forces, it is observed that the strength of the convective flow is promoted by an increase in \( N \). This is because increasing \( N \), for a fixed value of \( R_T \) (that is, a given strength of the thermal buoyancy forces) increases the total buoyancy forces. The effect of the magnetic field is depicted in fig.2

![Figure 2: Influence of magnetic field and thermal Rayleigh number on the flow intensity for \( Le = 10, R = 0.2, N = 1, b = \frac{15}{46} \)](image)

It is observed that as \( M \) increases progressively the flow intensity decreases. Even at such decrease, increasing thermal Raleigh number, \( R_T \), adds to the strength of the flow. Further considerations of the numerical solution shows that at \( M \geq 7, R_T = 20 \), the flow solutions are imaginary. Increasing the Soret,\( S \), decreases the flow intensity such that at \( M = 6 \), the flow is at the rest state for \( S \geq 0 \) and almost at the rest state for \( S = -0.5 \).
Figure 3: Influence of magnetic field and Soret on the flow intensity for $Le = 10, R = 0.2$, $N = 1, R_T = 10$ and $b = \frac{15}{16}$.

Figure 4 shows that increase in radiation increases the flow intensity. Increase also in the thermal Rayleigh number further enhances the flow intensity. It is observed from table 1 that increase in magnetic field decreases the rate of heat transfer and mass transfer. Increase in the thermal Rayleigh number is observed to further increase rate of mass transfer. For $M = 5$, the heat transfer is almost by pure conduction ($Nu \approx 1$).

4.2. Heat Driven Flow Limit

Here the thermal buoyancy forces and the Soret effect in the vicinity of the vertical boundaries induce a weak flow. The flow intensity for the clockwise and counterclockwise solutions are equal and both show decrease as $M$ increases. Table 2 shows that flow pattern becomes weaker as $M$ increases. This effect is such that at $M = 2$ when $R_T = 10$ circulation becomes completely impossible. Though the table is computed based on $R_T = 10$, an increase in the thermal Rayleigh number ensured an improvement on the strength of the flow and consequently enhanced flow possibilities (for example, at $M = 2$, $R_T = 70$, flow is possible but at $M = 6$, $R_T = 70$, flow is not possible). Such improvement by $R_T$, in the strength of the circulation also aids the rate of heat and mass transfer even when the magnetic field is hindering both. Our numerical computation shows that solution for the stream function and heat and mass transfer rate are independent of Soret at each $R_T$ value but not the same for the mass transfer. Increase in Soret increases mass transfer rate.

### Table 1

<table>
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<tr>
<th>$R_T$</th>
<th>$M$</th>
<th>$\Psi_0$</th>
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<th>$Sh$</th>
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**TABLE 1:** Influence of thermal Rayleigh number and magnetic field on heat and mass transfer for $Le = 10$, $S = 0.25, R = 0.2$, $N = 1$ and $b = \frac{15}{16}$.
Figure 5 shows that the heat transfer decreases considerably as $M$ increases. Hence for $M = 2$ the heat transfer is by pure conduction ($Nu \approx 1$). The effect of the magnetic field (figure 6) is much reduced on the mass transfer because of the more pronounced influence of the thermal Rayleigh number and the concentration distribution induced by the Soret and by convection.

For this pure thermal case, increase in radiation increases the flow intensity but decreases the heat transfer rate (Table 3). Increase in Soret has no effect on the heat transfer rate but significantly increases the mass transfer rate at the same thermal Rayleigh number, $R_T$.

**TABLE 2**: Influence of magnetic field and Soret on the stream function for $Le = 10, R = 0.2, N = 0, b = \frac{15}{16}, R_T = 10$

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<tr>
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<th>Nu</th>
<th>Sh</th>
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**TABLE 3**: Influence of Soret on the stream function and heat and mass transfer for $N = 0, Le = 10, M = 0.5, R_T = 20$ and $b = \frac{15}{16}$

<table>
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<td>4.00182</td>
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<td>3.98622</td>
<td>1.01043</td>
<td>11.708</td>
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</table>

**4.3. Opposing Double Diffusive Flow**

Five solutions are possible for the stream function – two clockwise ($\Psi_{0.1}$ and $\Psi_{0.2}$), two counterclockwise ($\Psi_{0.3}$ and $\Psi_{0.4}$) and the rest state. A symmetry runs on the solutions obtained such that $|\Psi_{0.1}| = |\Psi_{0.3}|$ and $|\Psi_{0.2}| = |\Psi_{0.4}|$. Because of similarity in behavior and also that the flow intensity of $\Psi_{0.1}$ and $\Psi_{0.4}$ are higher than that of $\Psi_{0.2}$ and $\Psi_{0.3}$, we shall emphasize more on $\Psi_{0.1}$ and $\Psi_{0.4}$.

**TABLE 4**: Influence of Soret, thermal Rayleigh number, radiation and buoyancy on the flow intensity for $Le = 10, b = \frac{15}{16}$ and $M = 0.5$

<table>
<thead>
<tr>
<th>S</th>
<th>$R_T$</th>
<th>R</th>
<th>$\Psi_{0.1}$</th>
<th>$\Psi_{0.4}$</th>
<th>$\Psi_{0.1}$</th>
<th>$\Psi_{0.4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.5</td>
<td>40</td>
<td>0</td>
<td>-1.25113</td>
<td>1.25113</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>-1.50811</td>
<td>1.50811</td>
<td>--</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-3.46699</td>
<td>3.46699</td>
<td>--</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>-4.99131</td>
<td>4.99131</td>
<td>--</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>-6.55405</td>
<td>6.55405</td>
<td>--</td>
<td>--</td>
<td></td>
</tr>
</tbody>
</table>
It is observed from Table 4 that the flow intensity increases with radiation. Increases in Soret and thermal Rayleigh number further enhances the flow intensity. Our numerical computation also shows that as the buoyancy $N$, is progressively decreased, the thermal buoyancy forces and the Soret effect prevailing in the vicinity of the vertical boundary induces a weak flow. In fact when $S = -0.5$, $R_T = 40$ and $N = -2$, convection is not possible. However by increasing the thermal Rayleigh number, $R_T$, we expect a multiplicity of solutions.

### TABLE 5: Influence of Soret and radiation on heat and mass transfer for $N = -1, M = 0.5, Le = 10$ and $b = \frac{15}{16}$

<table>
<thead>
<tr>
<th>$S$</th>
<th>$R_T$</th>
<th>$\Psi_{0,1}$</th>
<th>$Nu$</th>
<th>$Sh$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.5</td>
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<td>1.60172</td>
<td>4.16472</td>
</tr>
<tr>
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<td>0.5</td>
<td>-1.50811</td>
<td>1.57282</td>
<td>4.16459</td>
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<tr>
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<td>-3.46699</td>
<td>1.20493</td>
<td>4.07843</td>
</tr>
<tr>
<td>-0.5</td>
<td>3</td>
<td>-4.99131</td>
<td>1.10833</td>
<td>4.04485</td>
</tr>
<tr>
<td>-0.5</td>
<td>5</td>
<td>-8.13329</td>
<td>1.04312</td>
<td>4.01888</td>
</tr>
<tr>
<td>0.5</td>
<td>40</td>
<td>-7.72996</td>
<td>2.05063</td>
<td>4.16254</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>-1.99567</td>
<td>1.9236</td>
<td>4.15505</td>
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<tr>
<td>0.5</td>
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<td>4.07859</td>
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<tr>
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<td>3</td>
<td>-6.19309</td>
<td>1.6484</td>
<td>4.04664</td>
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<tr>
<td>0.5</td>
<td>5</td>
<td>-10.0582</td>
<td>1.06564</td>
<td>4.02023</td>
</tr>
</tbody>
</table>

Both $\Psi_{0,1}$ and $\Psi_{0,4}$ gives the same Nusselt and Sherwood numbers consequently it is observed that increased radiation increases the heat transfer rate and decreases the mass transfer rate but increasing the Soret coefficient slightly improves on the heat transfer but significantly improves the mass transfers (Table 5).

### 5. CONCLUSION

(i) For positive values of $N$, the strength of the convective flow is promoted by an increase in $N$. As radiation increases so is the flow intensity. Increase radiation also increase the rate of heat and mass transfer, whose magnitude increase as the Soret increase but more significantly on the mass transfer. As magnetic field increases progressively the flow intensity and the rate of heat and mass transfer decreases at a given thermal Rayleigh number, $R_T$. Despite this, further increase in the thermal Rayleigh number will improve on the flow intensity.

(ii) For the pure thermal convection, an increase in the thermal Rayleigh number, $R_T$, ensures an improvement on the strength of the clockwise and counterclockwise circulation and also on the rate of heat and mass transfer under action of increased radiation and magnetic field. The solutions reported here for $\Psi_{0}, Nu$ are independent of Soret at the same $R_T$ value. Increase in Soret increases mass transfer rate.

(iii) For negative values of $N$, the flow intensity increase with radiation, Soret, and thermal Raleigh number, while increasing the magnetic field and buoyancy weakens the flow. The Soret shows improvement in both heat and mass transfer but at varying rates. The magnetic field decreases heat transfer but induces little increase in mass transfer except where the Soret is increased to make such increase in mass transfer more remarkable.

### 6. REFERENCES


