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MATHEMATICAL MODELING FOR PERIODIC CAPACITATED ARC ROUTING PROBLEM

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ABSTRACT

The objective of this paper is to present the main mathematical models for periodic capacitated arc routing problems, the applications and the differences. These problems consist of determining a set of routes in a given network over a planning period and the number of times that a certain arc should be played according to a hierarchy over the planning horizon. It proposes mathematical modeling for a Multiple Tasks Periodic Capacitated Arc Routing Problem, this is each arc to be visited can run one of the possible tasks required for the day specified over the planning horizon.

Keywords: ***Mathematical modeling, routing, multiple tasks***

1. INTRODUCTION

The literature presents two classes of problems routing of vehicle, node routing and arc routing. The node routing problems consist of finding one or more routes to visit some or all nodes in a network representing the locations to be visited. Routing problems with arcs are intended to determine one or many routes to meet some or all of the arcs or edges of a network.

The well-known capacitated arc routing problem (CARP), consists of that each vehicle in a fleet has a certain capacity, and those vehicles must go through the arcs collecting or delivering certain demands without exceeding it. This problem is proposed by [1].

Applications for arc routing problems include: meter reading, snow and/or ice control, postal delivery, waste disposal, three-track inspection, auscultation instrumentation monitoring, etc.

Many applications generally require that the routes are planned in a multi-period horizon, which is a new problem called periodic CARP or PCARP defined by [2] .

The PCARP is a natural extension of the CARP when the routes are planned not just for a day, but for a set of days. It corresponds to various applications: urban waste collection, power lines inspection, rail inspection, road inspection for spreading snow in winter, inspection of auscultation instruments of dams, etc.

The mathematical modeling proposed for the PCARP is set in the context of inspection of auscultation instruments of dams to prevent accidents. The network monitoring is performed periodically. The galleries to be visited depend on the monitoring requirements, this requirement is in accordance with the need to read the instruments along the galleries, that is the reading frequency of each of the instruments. Each type of instrument installed in the gallery is considered as one type of task to be performed by a certain meter reader.

The paper is organized as follows. Initially we present a literature review of related work. In the next section is presented the main models of literature and its applications. The proposed model for the inspection of auscultation instruments problem is presented in the next section and some numerical results. The last section is devoted to the final considerations.

2. LITERATURE REVIEW:

According to [3] periodic capacitated arc routing problems are most deprived and disorganized that the periodic routing problems which are more studied, but has gained significant importance in scientific academic sense, because many applications in the real context of accident prevention are based on this type of problem.

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In the PCARP, as defined by [2] is; Given a network $G = (N, A)$ and a period of discreet planning H of s periods or days, in which each task (required arc) u has a number of services $f(u)$. This means that task must be u times, $1 \leq f(u) \leq s$, but at most once per day, treated, during the planning period H. The total number of services z to be performed in H is then the sum of all task services. The PCARP consists of finding a set of routes at minimum cost, that begins and ends from the deposit, satisfying the required number of services on all arcs and without exceeding the vehicle capacity.

The PCARP, according to [4], all arcs have a steady frequency requirement across the whole planning horizon. The task u must be treated f(u) times at each of the s periods or days. However, there are problems which are not regular in this sense as proposed by [5]. The authors did not present a mathematical model, however developed computational results by means of algorithms.

[6] were the first to propose a linear programming model for PCARP. In this modeling we are not aware of the possible periods of service set, for that consider the minimum and maximum spacing between the task. They studied the lower limits and a formal proof is presented, proving that the CARP lower limit is also a lower limit for the PCARP. A taboo search method was developed and applied to PCARP adapted in cases from the reference CARP.

In [7] they proposed a linear programming model with an approach to determine for day visits in the arcs according to the possible combinations of days allowed. They developed two heuristics, an insertion heuristic and a two-phase heuristic.

In [8] combined an algorithm of ant colony with an insertion heuristic to solve the PCARP achieving robust results and with a fast performance, did not present a mathematical model.

In [9] developed a memetic algorithm to solve a PCARP. The authors proposed a mathematical model where the objective function is composed of a primary objective and a secondary objective. In this model, the primary objective is to minimize the number of vehicles on the time horizon, and the secondary objective is the total cost.

In [5] is proposed a mathematical model and a two-phase algorithm to solve the problem in the first phase, the grouping of the arcs is done without violating the capacity of the route and in the second the routing, solving the routing problem of a single vehicle. For this modeling is not considered that the arcs follow the same regularity of visits as defined in [4]. Thus, one might want some arcs to be serviced twice during the first five days and once during the weekend, and day service may vary from one week to another. This type of problem was named as PCARP with irregular services (PCARP-I).

In [10] is presented a mathematical model for the routing problem in periodic trained for monitoring railroad tracks arcs. He includes in his model a penalty if the vehicle does not meet the arc or experience any delay, and he considered as vehicle capacity the fact that you can go and meet only a passage for the day appointed. He made a comparison between the models proposed in the literature and also a systemic evaluation of the proposed model by [5] and the proposed by the author.

3. MAIN MATHEMATICAL MODELS:

In [6] the first mathematical model considers the need for urban waste collection.

The decision variables used in this linear programming model are: $x_{ijvp} = 1$ iff edge (i,j) is traversed from i to j by vehicle v on day p, $l_{ijvkp} = 1$ iff task (i,j) is serviced from i to j by vehicle v on day p of combination k, and $Z_{ijk} = 1$ iff task (i,j) uses combination k, 0 otherwise.
Each permitted combination in $comb\ (i,j)$ is a set of combination indices. Denoted by $A_{kp} = 1$ iff the combination k contains day p. For each task (i,j). By convention, it is also defined for Q_{ijkp} the demand of the day p in arc (i,j) still $Q_{ijkp} = 0$ if and only if $k \notin comb\ (i,j)$ or $A_{kp} = 0$. The set $E_R(S)$ denotes the set of required edges whose both end nodes belong to a given subset of nodes S. In this formulation each edge and bow is a task to be performed a number of times over the planning horizon. And, considers W as the vehicular capacity.

The objective function is given by:

$$z = Min \sum_{(i,j)\in E} \sum_{v=1}^{V} \sum_{p=1}^{P} C_{i,j}\left(x_{ijvp} + x_{jivp}\right) \qquad (1)$$

Subject to the following constraints:

$$\sum_{k\in comb(i,j)} Z_{ijk} = 1 \qquad \{\forall\ (i,j) \in R \qquad (2)$$

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$$\sum_{(i,j)\in R} x_{ijvp} = \sum_{(j,i)\in R} x_{jivp} \qquad \begin{cases}\forall\, i \in X\\ \forall\, v = 1,\cdots,V\\ \forall\, p = 1,\cdots,P\end{cases} \tag{3}$$

$$x_{ijvp} \geq \sum_{k\in comb(i,j)} l_{ijvkp} \qquad \begin{cases}\forall\, (i,j) \in R\\ \forall\, v = 1,\cdots,V\\ \forall\, p = 1,\cdots,P\end{cases} \tag{4}$$

$$x_{jivp} \geq \sum_{k\in comb(i,j)} l_{jivkp} \qquad \begin{cases}\forall\, (i,j) \in R\\ \forall\, v = 1,\cdots,V\\ \forall\, p = 1,\cdots,P\end{cases} \tag{5}$$

$$\sum_{v=1}^{V} \left(l_{ijvkp} + l_{jivkp}\right) = A_{kp}.Z_{ijk} \qquad \begin{cases}\forall\, (i,j) \in R\\ \forall\, k \in comb(i,j)\\ \forall\, p = 1,\cdots,P\end{cases} \tag{6}$$

$$\sum_{(i,j)\in R} \sum_{k\in comb(i,j)}^{V} Q_{ijkp}\left(l_{ijvkp} + l_{jivkp}\right) \leq W \qquad \begin{cases}\forall\, v = 1,\cdots,V\\ \forall\, p = 1,\cdots,P\end{cases} \tag{7}$$

$$\sum_{i\in S} \sum_{j\notin S} x_{ijvp} \geq\, l_{rsvkp} + l_{srvkp} \qquad \begin{cases}\forall\, S \subseteq \{2,\cdots,n\}\\ \forall (r,s) \in E_R(S)\\ \forall\, p = 1,\cdots,P\\ \forall\, k \in comb(r,s)\\ \forall\, v = 1,\cdots,V\end{cases} \tag{8}$$

$$l_{ijvkp}\,, l_{jivkp}\,, x_{ijvp}\,, x_{jivp}\,, Z_{ijk} \in \{0,1\} \qquad \begin{cases}\forall\, (i,j) \in R\\ \forall\, v = 1,\cdots,V\\ \forall\, p = 1,\cdots,P\end{cases} \tag{9}$$

The objective function (1), to be minimized, is the total cost of the trips. Constraints (2) mean that one and only one day combination must be assigned to each task. Constraints (3) ensure trip continuity. In constraints (4) and (5), a task cannot be serviced if it is not traversed. In constraints (6), a task is serviced in day p iff this day belongs to the combination selected for the task. Moreover, the task is serviced by one single vehicle and in one direction only.

In constraints (7), vehicle capacity is respected for each day p and each trip v. This deserves some explanations. Consider one edge $[i,j]$. If it is not serviced on day p, then for each combination k, $l_{ijvkp} + l_{jivkp} = 0$ according to (6) and no quantity is counted for it. If the edge is serviced on day p, then from (6), there exists a day combination k such that $l_{ijvkp} + l_{jivkp} = 1$ and, since only one day combination k is assigned to it according to (3), the correct Q_{ijkp} will be counted.

In constraints (8), if $l_{ijvkp} + l_{jivkp} = 1$ then the vehicle v must traverse the cut at least once $(S, X/S)$, since both end-nodes of the required edge $[r,s]$ are in S. This does not prevent the formation of isolated cycles constituted only by deadheading edges. Fortunately, since the costs are non negative, it will never be the case in an optimal solution. All variables are binary thanks to (8).

In [7] is proposed a new model for collection of municipal waste. In this modeling is not constructed set of allowed combinations, but set the minimum and maximum spacing between each treatment. According to the spacing collections may differ from one service to another. Is denoted by $\Delta\min(i,j)$ and $\Delta\max(i,j)$ the minimum and maximum spacing, respectively, the daily production by q_{ijp} for each day p and by C vehicles capacity.

The decision variables used in this linear programming model are: $x_{ijvp} = 1$ iff edge $[i,j]$ is traversed from i to j by vehicle v on day p, $l_{ijvp} = 1$ iff task $[i,j]$ is serviced from i to j by vehicle v on day p, and $W_{ijvp}^{r} = 1$ iff daily production of $[i,j]$ in day r is collected by vehicle v on day p, and 0 otherwise. Is defined by $f_{i,j}$ the number of times that the task (i,j) must be met over the planning horizon.

$$z = Min \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{v=1}^{V} \sum_{p=1}^{P} c_{ij} x_{ijvp} \tag{10}$$

Subject to the following constraints:

$$\sum_{j=1}^{n} x_{ijvp} = \sum_{j=1}^{n} x_{jivp} \qquad \begin{cases}\forall\, v = 1,\cdots,V\\ \forall\, p = 1,\cdots,P\\ \forall\, i \in X\end{cases} \tag{11}$$

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$$x_{ijvp} + x_{ijvp} > l_{ijvp} \qquad \begin{cases} \forall\, v = 1, \cdots, V \\ \forall\, p = 1, \cdots, P \\ \forall\, [i,j] \in R \end{cases} \qquad (12)$$

$$\sum_{v=1}^{V} l_{ijvp} \leq 1 \qquad \begin{cases} \forall\, p = 1, \cdots, P \\ \forall\, [i,j] \in R \end{cases} \qquad (13)$$

$$\sum_{p=1}^{P} \sum_{v=1}^{V} l_{ijvp} = f_{ij} \qquad \begin{cases} \forall\, [i,j] \in R \\ \forall\, r = 1, \cdots, P \end{cases} \qquad (14)$$

$$\sum_{p=1}^{P} \sum_{v=1}^{V} W_{ijvp}^{r} = 1 \qquad \begin{cases} \forall\, [i,j] \in R \\ \forall\, r = 1, \cdots, P \end{cases} \qquad (15)$$

$$W_{ijvp}^{r} \leq l_{ijvp} \qquad \begin{cases} \forall\, v = 1, \cdots, V \\ \forall\, r, p = 1, \cdots, P \\ \forall\, [i,j] \in R \end{cases} \qquad (16)$$

$$W_{ijvp}^{r} \geq l_{ijvp} - \sum_{v=1}^{V} \sum_{k=r}^{P} W_{ijvk}^{r} - \sum_{v=1}^{V} \sum_{k=1}^{P-1} W_{ijvk}^{r} \qquad \begin{cases} \forall\, v = 1, \cdots, V \\ \forall\, p = 1, \cdots, P \\ \forall\, r = p+1, \cdots, P \\ \forall\, [i,j] \in R \end{cases} \qquad (17)$$

$$W_{ijvp}^{r} \geq l_{ijvp} - \sum_{v=1}^{V} \sum_{k=r}^{P-1} W_{ijvk}^{r} \qquad \begin{cases} \forall\, v = 1, \cdots, V \\ \forall\, p = 1, \cdots, P \\ \forall\, r = 1, \cdots, P \\ \forall\, [i,j] \in R \end{cases} \qquad (18)$$

$$\sum_{[i,j] \in R}^{V} \sum_{r=1}^{P} q_{ijr} W_{ijvk}^{r} \leq C \qquad \begin{cases} \forall\, v = 1, \cdots, V \\ \forall\, p = 1, \cdots, P \end{cases} \qquad (19)$$

$$\sum_{k=p}^{p+\Delta_{min}(i,j)-1} l_{ijvp} \leq 1 \qquad \begin{cases} \forall\, v = 1, \cdots, V \\ \forall\, p = 1, \cdots, P - \Delta_{min}(i,j) + 1 \\ \forall\, [i,j] \in R \end{cases} \qquad (20)$$

$$\sum_{k=p}^{p+\Delta_{max}(i,j)-1} l_{ijvp} \geq 1 \qquad \begin{cases} \forall\, v = 1, \cdots, V \\ \forall\, p = 1, \cdots, P - \Delta_{max}(i,j) + 1 \\ \forall\, [i,j] \in R \end{cases} \qquad (21)$$

$$\sum_{i \in S} \sum_{j \notin S} (x_{ijvp} + x_{jivp}) \geq l_{rsvp} \qquad \begin{cases} \forall\, p = 1, \cdots, P \\ \forall\, v = 1, \cdots, V \\ \forall\, S \subseteq \{1, 2, \cdots, n\} \\ \forall\, [r,s] \in R\colon\, r, s \in S \end{cases} \qquad (22)$$

$$l_{ijvp}, x_{ijvp}, x_{jivp}, W_{ijvp}^{r} \in \{0, 1\} \qquad \begin{cases} \forall\, p = 1, \cdots, P \\ \forall\, v = 1, \cdots, V \\ \forall\, r = 1, \cdots, P \\ \forall\, [i,j] \in R\colon \end{cases} \qquad (23)$$

The objective function (10), to be minimized, is the total cost of the trips. Constraints (11) ensure trip continuity. In constraints (12), a task cannot be serviced if it is not traversed. Constraints (13) require that in each day each task is serviced at most once by a single vehicle. Constraints (14) guarantee the necessary number of collections for each task. Constraints (15) ensure that the production of one day is collected by one single trip and in one direction only. Constraints (16) require that a daily production should not be collected on a day if there is no collection on that day. Constraints (17) and (18) say that there is no other collection between day r and day p if the daily production of day r is collected in day p for cases $r > p$ and $r \leq p$, respectively. Constraints (19) make sure that vehicle capacity is respected for each day p and each trip v. Constraints (20) and (21) ensure that the minimum and maximum spacing constraints between two successive treatments for one task are

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respected. Constraints (22) prevent invalid sub tours. All variables are binary thanks to (23).

In [5] is proposed a model for a periodic routing problem in qualified arcs, which function is to maximize the number of arcs met over the planning horizon, satisfying the requirements of intervals between each of the tasks. In this model the arcs are defined by class, since it is a road monitoring problem to dump salt in harsh winter, therefore some roads may require a greater number of visits over the weekend and less visits during the week.
For the formulation of the model it is considered the following sets:

Ω_c: set of arcs of class c

T: set of sets Ω_c

e_c: set of sub-periods (sub-horizons) associated to arcs of class c

$O(n)$: set of arcs leaving node $n \in N$

$I(n)$: set of arcs entering node $n \in N$

S: sub-set of arcs

$N(S)$: set of nodes incident to the arcs of S

V: set of routes (shifts)

W_j: set of routes corresponding to sub-period j

f_j: number of required services in sub-period j

p_a: weight of arc a.

The decision variables are $x_k^a = 1$ if arc $a \in R$ is serviced by route k; 0 otherwise, and y_k^a is the number of times arc a is traversed by the route k (without servicing). Each arc has two costs, C_a, the cost incurred on servicing arc a and t_a, the cost incurred on traversing arc a without servicing

The mathematical formulation is given by:

$$z = max \sum_{k\in V} \sum_{a\in R} p_a x_a^k \qquad (24)$$

$$\sum_{a\in O(n)} (x_a^k + y_a^k) - \sum_{a\in I(n)} (x_a^k + y_a^k) = 0 \qquad \{\forall k \in V, \quad n \in N \qquad (25)$$

$$\sum_{a\in R} c_a x_a^k + \sum_{a\in A} t_a y_a^k \le Q \qquad \{\forall\, k \in V \qquad (26)$$

$$M \sum_{a\in I(N(S))} (x_a^k + y_a^k) \ge \sum_{a\in O(N(S))} (x_a^k + y_a^k) \qquad \begin{cases} \forall S \subset A \\ 1 \notin N(S) \\ k \in V \end{cases} \qquad (27)$$

$$\sum_{k\in w_j} x_a^k \ge f_j \qquad \begin{cases} \forall\, j \in e_c, a \in \Omega_c \\ c = 1, \cdots, |T| \end{cases} \qquad (28)$$

$$x_a^k \in \{0; 1\}\ e\ y_a^k \in Z^+ \qquad \forall\, a \in R, k \in V \qquad (29)$$

The objective function (24) maximizes the services taking into account the class weight of arcs, those with higher importance having bigger weight. The set of constraints in (25) is the classical network flow conservation constraints. Route capacity is ensured by constraints (26). Note that the total capacity used by any route is obtained by adding the c_a and t_a of the arcs used. Constraints (27) are the connectivity constraints. M is a number equal to or greater than the maximum number of times an arc may be traversed. If an arc in S is traversed or serviced, at least one entering arc to set $N(S)$ must be traversed or serviced. The constraints related to the number of services per each period per class of arcs are given in (28). Finally, constraints (29) define the variables.

The mathematical models for the periodic capacitated arc routing problems do not deal with multiple tasks, the literature on the model to be proposed comes to problems with this feature.

4. THE PROPOSED MATHEMATICAL MODEL:

The proposed modeling is inspired by an inspection of auscultation instruments of dams. In general, dams follow a strict monitoring plan, which the meter readers must periodically go through the galleries to collect data from the instruments. In order to establish a criterion of standardization of readings optimizing the route, it was sought the formulation of a mathematical model of linear programming.

Set the three-dimensional matrix to represent the combinations permitted given by $a_{d,\Psi,k} = 1$ if the day d belong to combination Ψ in relation to the instrument k.

The decision variables, $m_{i,j,\Psi,k}$ if the edge (i,j) is chosen at the combination ψ in relation to the instrument k takes value 1. Otherwise zero; $w_{i,j,d,k}$ if the edge (i,j) is visted on day d in relation to the

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instrument k takes value 1. Otherwise zero; $l_{i,j,v,k,d}$ if the edge (i,j) is attended by the meter reader v on day d for data collection of the instrument type k assumes value 1. Otherwise zero; $x_{i,j,v,d}$ counts the number of times the edge (i,j) it is traveled by the meter reader v on day d; $z_{v,d,k}$ counts the number of equipments used on day d by the meter reader v.

The set of input is given by, $d_{i,j}$ distance between nodes i and j; A_{p_k} quantity of reading equipment's of the instrument type k; CL_v carrying capacity for the meter reader v; P_{kv} associated weight to the reading equipment type k in relation to the meter reader v; N_{TA} number of total distinct instruments; $|E|$ number of arcs that compose the graph; γ_k time to read the tool type k; $q_{k,i,j}$ quantity of instruments installed on the edge (i,j); $V_{m,v}$ average speed of the meter reader v; W_v capacity of working hours for the meter reader v.

The mathematical formulation is given to minimizing z given by

$$z = min \sum_{i=1}^{N_V} \sum_{j=1}^{N_V} \sum_{d=1}^{D} \sum_{v=1}^{NV} d_{ij}.x_{i,j,v,d} \qquad (30)$$

Subject to the following constraints

$$\sum_{\psi \in \Psi_K} (m_{i,j,\psi,k} + m_{j,i,\psi,k}) = 1 \qquad \begin{cases} (i,j) \in E \\ k = 1, 2, \cdots, N_{TA} \end{cases} \qquad (31)$$

$$w_{i,j,d,k} = \sum_{\psi \in \Psi_K} m_{i,j,\psi,k} \,.\, a_{d,\psi,k} \qquad \begin{cases} d = 1, 2, \cdots, D \\ (i,j) \in E \\ k = 1, 2, \cdots, N_{TA} \end{cases} \qquad (32)$$

$$\sum_{v=1}^{NV} z_{v,d,k} \le A_{p_k} \qquad \begin{cases} k = 1, 2, \cdots, N_{TA} \\ d = 1, 2, \cdots, D \end{cases} \qquad (33)$$

$$\sum_{k=1}^{N_{TA}} P_{k,v}.z_{v,d,k} \le CL_v \qquad \begin{cases} v = 1, 2, \cdots, NV \\ d = 1, 2, \cdots, D \end{cases} \qquad (34)$$

$$\sum_{v=1}^{NV} l_{i,j,v,d,k} \le w_{i,j,d,k} \qquad \begin{cases} e_{i,j} \in E \;\; ou \; a_{i,j} \in A \\ k = 1, 2, \cdots, N_{TA} \\ d = 1, 2, \ldots, D \end{cases} \qquad (35)$$

$$N_{TA}.x_{i,j,v,d} = \sum_{k=1}^{N_{TA}} l_{i,j,v,d,k} \qquad \begin{cases} e_{i,j} \in E \;\; ou \; a_{i,j} \in A \\ v = 1, 2, \cdots, NV \\ d = 1, 2, \ldots, D \end{cases} \qquad (36)$$

$$\sum_{i=2}^{N_V} x_{1,i,v,d} \le 1 \qquad \begin{cases} d = 1, 2, \ldots, D \\ v = 1, 2, \ldots, NV \end{cases} \qquad (37)$$

$$\sum_{j=1}^{N_V} x_{j,i,v,d} = \sum_{j=1}^{N_V} x_{i,j,v,d} \qquad \begin{cases} i = 1, 2, \ldots, N_V \\ v = 1, 2, \cdots, NV \\ d = 1, 2, \ldots, D \end{cases} \qquad (38)$$

$$|E| z_{v,d,k} \ge \sum_{i=1}^{N_V} \sum_{j=1}^{N_V} l_{i,j,v,d,k} \qquad \begin{cases} k = 1, 2, \ldots, N_{TA} \\ v = 1, 2, \cdots, NV \\ d = 1, 2, \ldots, D \end{cases} \qquad (39)$$

$$|E| \sum_{j=2}^{N_V} x_{1,j,v,d} = \sum_{i=2}^{N_V} \sum_{j=2}^{N_V} x_{i,j,v,d} \qquad \begin{cases} d = 1, 2, \ldots, D \\ v = 1, 2, \ldots, NV \end{cases} \qquad (40)$$

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$$\sum_{k=1}^{N_{TA}} \sum_{(i,j)\in E} \gamma_k . q_{k,i,j} . l_{i,j,v,d,k} + \sum_{(i,j)\in E} \frac{d_{i,j}}{V_{m,v}} . x_{i,j,v,d} \leq W_v \quad \begin{cases} d = 1,2,\dots,D \\ v = 1,2,\dots,NV \end{cases} \quad (41)$$

$$\sum_{r\in Q} \sum_{s\notin Q} x_{r,s,v,d} \geq \frac{1}{|Q|^2 - |Q|} \sum_{i,j\in Q} x_{i,j,v,d} \quad \begin{cases} v = 1,2,\cdots,NV \\ d = 1,2,\dots,D \\ \forall\, Q \subset \{2,3,\dots,n\} \\ |Q| \geq 2 \end{cases} \quad (42)$$

$$x_{i,j,v,d} \in \mathbb{Z}_+ \quad \begin{matrix} l_{i,j,d,k};\ t_{v,d};\ z_{v,d,k} \in \{0,1\} \\ m_{i,j,\psi,k};\ w_{i,j,d,k} \in \{0,1\} \end{matrix} \quad (43)$$

The objective function given in (30) seeks to minimize the total distance traveled by the meter reader along the planning horizon.

The constraints (31) guarantees that only one combination will be designated for the reading of the instrument k installed at arc (i,j) and in only one direction.

The constraints (32) guarantees that, for each arc (i,j), the readings occur on the day wich has been designated, based on the chosen combination.

The constraints (33) guarantees that the quantity of equipments type k used by all the meter readers in one day d of reading be lower than the allowed quantities.

The constraints (34) guarantees that the capacity of the meter reader guarantees that the capacity of the meter reader is not extrapolated in relation to the equipments he carries to collect data in one day of reading.

The constraints (35) guarantees that the meter reader will travel the arc (i,j) for data collection of the instrument type k, if the arc is allocated for that day, satisfying the condition of combination allowed for that day.

The constraints (36) guarantees that the arc (i,j) will be traveled by the meter reader v on day d if he has any equipment and the arc have been designated for that day.

The constraints (37) guarantee that the meter reader be used once in the day and, if he has other equipments, that he will start the reading from the same arc.

The constraints (38) guarantee the flow's continuity.

The constraints (39) assures that the equipment type k will be carried by the meter reader v at the day d.

The constraints (40) guarantees that if any meter reader toured some arc, should leave the office.

The constraints (41) guarantees that the time to accomplish the readings will not go longer than the working hours for each meter reader.

The constraints (42) prevents the formation of isolated depositing sub cycle.

The constraints (43) guarantees that the variables are integer and assures that the variables are binary.

5. COMPUTATIONAL RESULT:

Consider the following chart according to the Figure 1 configuration, where node 1 is the office, the values in parentheses between two of the distinct vertices denote the arc length and the number of instruments installed, respectively, for all the tasks the number of instruments it will be installed.

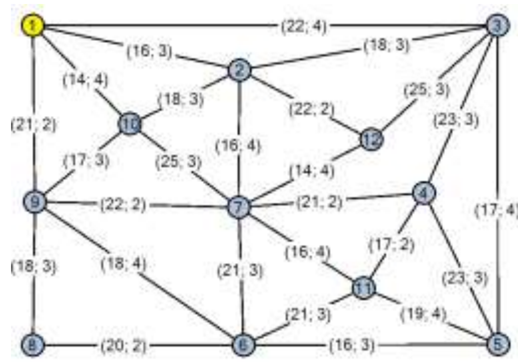

Figure 1: Case configuration

The planning horizon for each task will be 5 days, which comprises from Monday to Friday, and numbered in that order. The time that the meter reader takes to perform each task is given by 35, 40, 45, 50 seconds to each task respectively. The average walking speed for each meter reader is $1{,}35 m/s$ and time availability to perform the task and return to the office is 25200 seconds.

The possible combinations permitted to perform each of the tasks are presented in Chart 1.

TYPE OF TASK (K)	NUMBER OF READINGS	COMBINATION ψ_K	DAYS OF VISITS

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1 (piezometers)	2	1	(1, 3)
		2	(2, 4)
		3	(3, 5)
2 (seismographs)	3	1	(1, 2, 3)
		2	(2, 3, 4)
		3	(3, 4, 5)
3 (pendulums)	2	1	(1, 2)
		2	(2, 3)
		3	(3, 4)
		4	(4, 5)
4 (tensiometers)	2	1	(1, 4)
		2	(2, 5)

Chart 1: Permitted combinations

Name	Nº nod.	Nº Arc	Tasks	Nº Veh.	Nº Comb.	Time of Job	Simul. Tasks	Proposed model		
								GAP %	Fun. Obj.	Time-Seg
Gpr1	12	52	1-2	3	6	25200	2	7,06	1650	37021,27
Gpr2	12	52	1-3	3	10	25200	2	41,98	2296	208606,97
Gpr3	12	52	1-3	3	10	25200	3	34,95	1805	210772,38
Gpr4	12	52	1-4	3	12	25200	2	44,79	3244	79180,80
Gpr5	12	52	1-4	3	12	25200	3	14,96	2156	83000,50

Table 1: Computational results

The tests were done on a computer Intel(R) Xeon(R) CPU E5-2650, 2.00 GHz memory of 32 GB, 64-bit Windows.

The Table 1 presents the results of the model proposed in the following situations, the problem Grp1 the meter reader can perform the tasks 1 and 2 simultaneously. In the situation Grp2, of 3 possible tasks of the meter reader can perform 2 of them simultaneously. In the situation Grp3 of 3 possible tasks of the meter reader can perform 3 tasks simultaneously. In the situation Grp4 of 4 tasks the meter reader can perform two tasks simultaneously and the situation Grp5 of 4 tasks of the meter reader can perform 3 tasks simultaneously.

6. CONCLUSIONS

The studies in periodic capacitated arc routing problems are recent, there are few studies related to this subject. In general, the problems encountered are inspired by the collection of municipal waste. In Montroy (2013) and Batista (2014) the proposed model involves problem of accident prevention. However, in all jobs each arc is considered as a task to be performed with a certain frequency.

In this paper we proposed a mathematical model for periodic capacitated arc routing problems, the proposal is distinguished precisely by presenting multiplicity of tasks to be performed in each of the arcs.

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